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MATHEMATICS
IN
AGRICULTURE

by

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Preface

In recent years there has been an increasing demand for a practical book on mathematics designed especially to fit the needs of persons interested in agriculture. This book is the result of an earnest effort to meet that demand.

While the text is not meant to serve as an authoritative source on technical matter pertaining to agriculture, some technical matter has been included, in an effort to vitalize the subject by giving to problems their proper agricultural setting. Also, the tables near the end of the book contain information which possesses practical value for anyone engaged in agriculture.

The author feels indebted to many members of the teaching staff of the Agricultural and Mechanical College of Texas for valuable suggestions and criticisms regarding the content of this book. Most of the illustrations were prepared by Mr. J. G. McGuire and Mr. R. O. Loving of the Department of Engineering Drawing.

R. V. McGee

Contents

CHAPTER 1

Mathematical Operations

Fundamental Operations

The four main operations of mathematics are addition, subtraction, multiplication, and division. The result obtained by addition of numbers is called a *sum;* the result of subtraction is called a *difference;* the result of multiplication is called a *product;* and the result of division is called a *quotient.* Thus, the sum of the two numbers 10 and 2 is 12; their difference is 8; their product is 20; and, if 10 is divided by 2, the quotient is 5, while, if 2 is divided by 10, the quotient is $\frac{2}{10}$, or $\frac{1}{5}$.

Practically all steps taken in solving problems involve the above operations and no others. Phrases like "canceling numbers," "dropping numbers," and "transposing numbers" do not identify mathematical operations, and the student will do well to avoid using such phrases.

Abstract and Concrete Numbers

All numbers are abstract, in the sense that they have none of the properties of physical objects. However, a number expression that represents a quantity of some kind, as 5 lb. or 3 in., is usually referred to as a concrete number. A concrete number may be thought of as consisting of an abstract number times some unit of measure. In the expression *5 lb.*, 5 is abstract and indicates the number of times the unit, *pound*, is taken in measuring the weight of some object or the magnitude of some force.

Strictly speaking, arithmetical operations are performed with abstract numbers only. For instance, to find the total weight of two objects, one of which weighs 5 lb. and the

other 3 lb., we add 5 and 3 and get as a result the abstract number 8, and then conclude that the weight of the two objects together is 8 lb. However, in practice concrete numbers themselves are frequently connected by signs of addition, subtraction, and so on, as if the operations were to be performed with certain quantities instead of with the abstract numbers which are the measures of the quantities. Thus, we write 5 lb. + 3 lb. = 8 lb.

Whole Numbers, Fractions, and Mixed Numbers

Numbers like 6, 29, 17, 354, and so on, are called *whole numbers* or *integers*. All numbers used in counting are whole numbers.

A fraction is an indicated division of one number by another. Thus $\frac{5}{8}$ is a fraction in which it is indicated that 5 (called the *numerator*) is divided by 8 (called the *denominator*). The student should recall the fundamental principle of fractions, which states that the value of a fraction is not changed if its numerator and denominator are multiplied or divided by the same number, not zero, division by zero being impossible. Practically all of the work done with fractions is based upon this principle.

A number composed of an integer and a fraction taken together is called a *mixed number*. A mixed number is actually an indicated sum. Thus $16\frac{2}{3}$ is a short way of writing the sum $16 + \frac{2}{3}$.

Decimal Numbers

The word *decem* is a Latin word meaning "ten." Our system of arithmetical numbers is called the decimal system because 10 is the basic number of the system. For instance, the number 356 means $300 + 50 + 6$, or $3(100) + 5(10) + 6(1)$. If the student recalls the meaning of a positive integral exponent, and of zero as an exponent, he will see that this number may also be written as $3(10^2) + 5(10^1) + 6(10^0)$. Thus each digit of a number is a multiplier of a certain power

of 10, the particular power depending upon the place that the digit occupies in the number. This decimal representation of numbers is extended further in the writing of mixed numbers and fractions by placing a period to the right of the 10^0 digit and considering the digits to the right of this period (or decimal point) as multiplying 10^{-1}, 10^{-2}, 10^{-3}, and so forth. It is well for the student to recall here the meaning given to a negative exponent. For example,

$$10^{-1} = \frac{1}{10}, \quad 10^{-2} = \frac{1}{10^2} = \frac{1}{100}, \quad 10^{-3} = \frac{1}{10^3} = \frac{1}{1000}, \quad \text{and so forth.}$$

The number 7356.482 may now be considered as a short way to write the sum

$$7(10^3) + 3(10^2) + 5(10^1) + 6(10^0) + 4(10^{-1}) + 8(10^{-2}) + 2(10^{-3}),$$

or

$$7000 + 300 + 50 + 6 + \tfrac{4}{10} + \tfrac{8}{100} + \tfrac{2}{1000}.$$

The names of the places most frequently used in writing decimal numbers are given in the following scheme:

Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths
7	3	5	6	.	4	8	2

This number may be read, "Seven thousand, three hundred fifty-six, and four hundred eighty-two thousandths." In practice, it would usually be read, "Seven, three, five, six, point, four, eight, two."

The following rules should be observed in performing operations with decimal numbers:

1. In addition or subtraction, write the numbers in a vertical column with the decimal points aligned and add or subtract as with whole numbers.

2. In multiplication, multiply as if the numbers were whole numbers; then, beginning at the right, point off as

many decimal places in the product as there are decimal places in all of the factors together.

3. In division, multiply both the divisor and the dividend by that power of 10 which will make the divisor an integer; divide as with whole numbers; and point off as many places in the quotient as there are in the dividend.

4. A common fraction may be changed to decimal form by dividing the numerator by the denominator, first placing a decimal point after the numerator and annexing as many zeros as desired. Common fractions (in lowest terms) whose denominators contain factors other than 2 and 5 cannot be expressed exactly as terminating decimals. Such fractions can be expressed decimally to a desired number of places.

The student will find the short methods of multiplication and division indicated below well worth learning.

1. To multiply a number by 10, 100, or 1000, move the decimal point to the right one, two, or three places, respectively, annexing zeros if necessary.
2. To multiply by 25, multiply by 100 and divide by 4.
3. To multiply by 50, multiply by 100 and divide by 2.
4. To multiply by 125, multiply by 1000 and divide by 8.
5. To multiply by 9, multiply by 10 and subtract the multiplicand.
6. To multiply by 11, multiply by 10 and add the multiplicand.
7. To divide by 10, 100, or 1000, move the decimal point to the left one, two, or three places, respectively, prefixing zeros if necessary.
8. To divide by 25, multiply by 4 and divide by 100.
9. To divide by 50, multiply by 2 and divide by 100.
10. To divide by 125, multiply by 8 and divide by 1000.

The student should see to it that he knows how to add, subtract, multiply, and divide with whole numbers, common fractions, mixed numbers, and decimal numbers; and he should strive to develop accuracy and speed in perform-

ing these operations. Reference to the table on pages 6-7 may prove helpful.

Problems*

1. Express the sum of 12 gal., 10 qt., and 13 pt. in gallons; in quarts; in pints.

2. 4 lb. 12 oz. + 3 lb. + 9 lb. 15 oz. + 13 oz. Express this sum (a) in pounds and ounces; (b) in ounces only; (c) in pounds only.

3. Add: $13.45, $0.80, $76, $9.75, 63¢.

4. $\frac{7}{8} + \frac{3}{4} - \frac{5}{6} = ?$

5. $65\frac{4}{9} - 37\frac{7}{12} = ?$

6. 480 ft. 2 in. − 197 ft. 10 in = ?

7. Multiply $20\frac{3}{4}$ bu. by 9.

8. 1 in. is what part of a foot? 7 in. is what part of a foot? 7 lb. is what part of 12 lb.?

9. 35 bu. is what part of 210 bu.?

10. $3\frac{1}{2}$ lb. is what part of $24\frac{1}{2}$ lb.?

11. 156 A. is what part of 390 A.?

12. A field containing $186\frac{3}{4}$ A. is divided into 9 equal plots. How many acres are there in each plot?

13. How many $2\frac{1}{2}$-bu. baskets of corn are there in a crib containing 30 bu.?

14. What is the weight of the water in a cistern that contains 464 cu. ft.?

15. How many bushels of wheat can be put in a bin that contains 900 cu. ft.?

16. How many gallons of water will it take to fill an aquarium containing $1\frac{1}{2}$ cu. ft.?

17. If a horse is 15 hands high, what is his height in inches?

18. A man can row across a lake $2\frac{3}{4}$ mi. wide in 42 min. What is his rate of rowing per hour?

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* Refer to the tables at the end of the book for information needed in solving problems.

Operations with Arithmetical Numbers

Kind of Numbers	Addition	Subtraction	Multiplication	Division
Whole Numbers	$87 + 356 + 9 = ?$ $\begin{array}{r} 87 \\ 356 \\ 9 \\ \hline 452 \end{array}$	$3805 - 1847 = ?$ $\begin{array}{r} 3805 \\ 1847 \\ \hline 1958 \end{array}$	$537 \times 48 = ?$ $\begin{array}{r} 537 \\ 48 \\ \hline 4296 \\ 2148 \\ \hline 25776 \end{array}$	$932 \div 37 = ?$ $\begin{array}{r} 25\frac{7}{37} \\ 37/\overline{932} \\ 74 \\ \hline 192 \\ 185 \\ \hline 7 \end{array}$
Common Fractions	$\frac{4}{9} + \frac{5}{6} = ?$ $\frac{4}{9} = \frac{8}{18}$ $\frac{5}{6} = \frac{15}{18}$ $\frac{23}{18} = 1\frac{5}{18}$	$\frac{7}{9} - \frac{5}{12} = ?$ $\frac{7}{9} = \frac{28}{36}$ $\frac{5}{12} = \frac{15}{36}$ $\frac{13}{36}$	$\frac{5}{12} \times \frac{3}{4} = ?$ $\frac{5}{\underset{4}{\cancel{12}}} \times \frac{\overset{1}{\cancel{3}}}{4} = \frac{5}{16}$	$\frac{5}{6} \div \frac{3}{8} = ?$ $\frac{5}{6} \div \frac{3}{8}$ $= \frac{5}{\underset{3}{\cancel{6}}} \times \frac{\overset{4}{\cancel{8}}}{3}$ $= \frac{20}{9} = 2\frac{2}{9}$
	(a) $8\frac{2}{3} + 17 = ?$ $\begin{array}{r} 8\frac{2}{3} \\ 17\phantom{\frac{2}{3}} \\ \hline 25\frac{2}{3} \end{array}$	(a) $43 - 19\frac{5}{8} = ?$ $43 = 42\frac{8}{8}$ $19\frac{5}{8} = 19\frac{5}{8}$ $23\frac{3}{8}$	(a) $12 \times 18\frac{3}{4} = ?$ $12 \times 18\frac{3}{4}$ $= 216 + 9 = 225$	(a) $6\frac{2}{5} \div 8 = ?$ $6\frac{2}{5} \div 8$ $= \frac{\overset{4}{\cancel{32}}}{5} \times \frac{1}{\underset{1}{\cancel{8}}} = \frac{4}{5}$

Mixed Numbers	(b) $8\frac{1}{6} + \frac{5}{8} = ?$ $8\frac{1}{6} = 8\frac{4}{24}$ $\frac{5}{8} = \frac{15}{24}$ $8\frac{19}{24}$	(b) $23\frac{5}{6} - 9\frac{3}{4} = ?$ $23\frac{5}{6} = 23\frac{10}{12}$ $9\frac{3}{4} = 9\frac{9}{12}$ $14\frac{1}{12}$	(b) $12\frac{1}{2} \times \frac{3}{5} = ?$ $12\frac{1}{2} \times \frac{3}{5}$ $= \frac{\overset{5}{\cancel{25}}}{2} \times \frac{3}{\underset{1}{\cancel{5}}}$ $= \frac{15}{2} = 7\frac{1}{2}$	(b) $7\frac{1}{2} \div \frac{3}{4} = ?$ $7\frac{1}{2} \div \frac{3}{4}$ $= \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}}$ $= \frac{10}{1} = 10$
	(c) $37\frac{3}{4} + 65\frac{5}{6} = ?$ $37\frac{3}{4} = 37\frac{9}{12}$ $65\frac{5}{6} = 65\frac{10}{12}$ $102\frac{19}{12}$ $= 103\frac{7}{12}$	(c) $73\frac{4}{9} - 26\frac{7}{12} = ?$ $73\frac{4}{9} = 73\frac{16}{36} = 72\frac{52}{36}$ $26\frac{7}{12} = 26\frac{21}{36} = 26\frac{21}{36}$ $46\frac{31}{36}$	(c) $6\frac{2}{5} \times 3\frac{1}{3} = ?$ $6\frac{2}{5} \times 3\frac{1}{3}$ $= \frac{32}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{3}$ $= \frac{64}{3} = 21\frac{1}{3}$	(c) $8\frac{3}{4} \div 6\frac{2}{3} = ?$ $8\frac{3}{4} \div 6\frac{2}{3}$ $= \frac{35}{4} \div \frac{20}{3}$ $= \frac{\overset{7}{\cancel{35}}}{4} \times \frac{3}{\underset{4}{\cancel{20}}}$ $= \frac{21}{16} = 1\frac{5}{16}$
Decimal Numbers	$38.7 + 7.625 + .09 + 15 = ?$ 38.7 7.625 .09 15. 61.415	$73.5 - 26.72 = ?$ 73.50 26.72 46.78	$18.36 \times 4.7 = ?$ 18.36 ×4.7 12852 7344 86.292	$20.992 \div 6.4 = ?$ 3.28 6.4/20.992 19 2 1 79 1 28 512 512

19. A man finds that his automobile goes $86\frac{1}{4}$ mi. on 5 gal. of gasoline. How many gallons of gasoline would he expect to use on a 354-mi. trip?

20. At $35 per acre what is the value of $87\frac{3}{4}$ A. of land?

21. Write in words the number 2086.354.

22. Write as a decimal the number twelve thousand, four hundred sixty-seven, and thirty-eight thousandths.

23. At $75 an acre how much will 62.4 A. of land cost?

24. Find the value of 48.6 tons of hay at $15.75 per ton.

25. A farmer has cotton planted on four separate plots of ground whose acreages are 34.6 A., 28.75 A., .8 A., and 19 A. What is the total acreage planted in cotton?

26. If 4386 lb. of lint are produced on 23.4 A., what is the yield per acre, expressed to the nearest hundredth of a pound?

27. Express the following as exact decimal fractions: $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{16}$, $\frac{2}{5}$.

28. Express the following as decimal fractions, to the nearest thousandth: $\frac{2}{3}$, $\frac{5}{6}$, $\frac{2}{7}$, $\frac{5}{9}$.

29. If the lengths of the four sides of a field are 22.75 rd., 17.65 rd., 28.6 rd., and 18.35 rd., what will the fence to inclose the field cost at $4.13 per rod?

30. If the posts for the fence mentioned in Problem 29 are placed not more than 12 ft. apart, about how many posts will be required?

31. In a certain fertilizer for corn land $.475\frac{5}{8}$ of the fertilizer is cottonseed meal, .43 is phosphoric acid, and the rest is kainit. How many pounds of each are there in a ton of the fertilizer?

32. A sample of average milk is found to be .87 water (by weight). How many pounds of water are there in 10 gal. of such milk?

33. A steer whose live weight was 560 lb. dressed 308 lb. The dressed weight was what part of the live weight? Express the result as a decimal.

34. 1760 lb. is what part of a ton? Express the result as a decimal.

35. A $12\frac{1}{2}$-lb. strip of bacon was bought at 28¢ per pound and sold sliced, without rind, at 50¢ a pound. What was the profit if the weight of the rind was .16 of the weight of the strip?

36. A certain grain mixture for poultry consists of 150 lb. of wheat, 200 lb. of cracked corn, and 100 lb. of oats. What part of this mixture is wheat? What part is cracked corn? What part is oats?

37. If barley is quoted at 80¢ per bushel, what is the value of a ton of barley?

38. If new ear corn is worth $28 per ton, what is the value per bushel?

39. If ear corn is worth $24 per ton on May 1 of a certain year, what should shelled corn be worth, neglecting shelling costs and value of cobs?

40. Which is the greater, one third of $18\frac{4}{7}$ or three times $2\frac{1}{14}$? How much greater?

41. If a steer whose dressed weight was 308 lb. lost $\frac{9}{20}$ of its live weight in dressing, what was its live weight?

42. In one year $\frac{9}{20}$ of the usual cotton acreage on a certain farm was planted in other crops. If there were 78 A. left in cotton, what was the acreage usually planted in cotton on this farm?

43. In a certain field that is fairly uniformly white with cotton, it is observed that the seed cotton picked from 4 rows weighs 120 lb. If the field measures 480 yd. (crosswise to the rows) and the rows are 3 ft. apart and of equal length, estimate the yield in bales from this picking of the field, counting 1500 lb. of seed cotton to the bale.

44. In a certain 1600-lb. bale of seed cotton (the remnant of the crop) the land owner, a tenant, and a cropper have interests as follows: The land owner has full interest in 300 lb., one-fourth interest in 400 lb., and half interest in 900 lb.; the tenant has three-fourths interest in 400 lb.; and the cropper has half interest in 900 lb. If ginning yields 540 lb. of lint that sells at 12¢ per lb. and 1000 lb. of seed that sells at $24 per ton, how should the receipts be divided among the three interested persons after $6 is deducted to cover ginning, storage, and insurance?

45. Light travels at approximately 186,330 mi. per second. Find to the nearest second how long it takes light to travel from the sun to the earth, a distance of approximately 92,897,400 mi.

46. It is estimated that 700 lb. of shelled corn will produce 100 lb. of live-weight pork, and that 300 lb. of shelled corn with 25 lb. of tankage and 25 lb. of cottonseed meal will produce 100 lb. of live-weight pork. If shelled corn is worth $1.25 per hundred-

weight, tankage is worth \$3 per hundredweight, and cottonseed meal is worth \$1.25 per hundredweight, would it pay a farmer to trade some of his corn for tankage and cottonseed meal and feed the mixture? How much can he afford to pay for tankage to make this replacement of corn by tankage?

47. The following mixture of feeds makes 1000 lb. of a standard baby chick ration: 300 lb. ground yellow corn at \$1.75 per hundredweight; 300 lb. gray wheat shorts at \$1.65 per hundredweight; 100 lb. ground oats at \$1.65 per hundredweight; 60 lb. dried buttermilk at \$7 per hundredweight; 60 lb. meat scraps at \$3 per hundredweight; 60 lb. cottonseed meal at \$1.75 per hundredweight; 50 lb. alfalfa leaf meal at \$2 per hundredweight; 20 lb. finely ground oyster shell at 85¢ per hundredweight; 10 lb. salt at \$1.10 per hundredweight; 40 lb. bran at \$1.40 per hundredweight; 1 pint triple-strength cod liver oil at \$1.25 per gallon. This same feed can be bought ready mixed at \$3.50 per hundredweight. Disregarding the labor cost of mixing and also the difference in thoroughness of mixing, find how much a farmer saves per 100 lb. by mixing the feed instead of buying it ready mixed.

48. A man employed on a dairy farm received for a year's work the following: \$50 a month as wages, free house rent worth \$25 per month, 365 gal. of milk worth 36¢ per gallon, 52 lb. of butter worth 30¢ per pound, and 12 bu. of potatoes worth \$1.80 per bushel. What he received was equivalent to what salary for the year?

49. What is the profit per quart on milk bought for 15¢ a quart and sold at 10¢ a glass, there being two glasses to a pint?

50. It is estimated that skim milk for feeding purposes is worth half as much per 100 lb. as corn is worth per bushel. What should be the value of 160 lb. of skim milk when corn is worth 90¢ per bushel?

51. On the basis of the comparison of feeding values mentioned in Problem 50, a pound of shelled corn is worth how many times as much as a pound of skim milk? The value of a pound of skim milk is what part of the value of a pound of shelled corn?

52. It is estimated that an 8-in. fan uses 20 w per hour.* At

* The unit of electrical energy is the watt-hour (w-hr.), which is the electrical energy capable of working at the rate of 1 w for 1 hr. A kilowatt-hour (kw-hr.) is 1000 w-hr. and is the unit that most electric meters register and upon which rates for electricity are usually based. Lamps and other electrical appliances usually bear markings that indicate the number of watts used per hour. Thus a lamp marked 75 w uses 75 w per hour.

4¢ per kw-hr., what would be the cost of running this fan 8 hr. each day for 26 da.?

53. At 3¢ per kw-hr. what should it cost to operate a 6-lb. 550-w flat iron for 40 hr.?

54. A certain city has the following rates for electric power: First 20 kw-hr. at 10¢, with a minimum charge of $2 per month; next 30 kw-hr. at 5¢; next 50 kw-hr. at 4¢; next 200 kw-hr. at 3¢; and all in excess of 300 kw-hr. at 2¢ per kilowatt-hour. Find the amount of the bill for each of the following monthly consumptions of electric power: (a) 18 kw-hr., (b) 35 kw-hr., (c) 70 kw-hr., (d) 210 kw-hr., (e) 450 kw-hr.

55. The following estimates were made of the electric energy required each month for a certain home: Lighting, 25 kw-hr.; iron 6 kw-hr.; refrigerator, 50 kw-hr.; radio, 10 kw-hr. Using the rates given in Problem 54, calculate the amount of the bill for the month.

56. Using the rates given in Problem 54, calculate the amount of the monthly bill for electric energy used in a home with the following estimated requirements:

Appliance	Number of Appliances	Watts per Hour Used by Appliance	Hours Used per Day	Days Used per Month
Lamp	1	25	2	30
Lamp	3	60	4	30
Lamp	2	75	3	30
Lamp	1	100	2	30
Iron	1	500	6	4
Washing machine	1	200	3	4
Refrigerator	1	250	8	30
Radio	1	150	6	30

57. Using the rates given in Problem 54, make an estimate of the cost per month of lighting a 5-room house, listing the requirements for each room.

58. At 3¢ per kilowatt-hour, what would be the cost of brooder operation for 1000 baby chicks for a period of 6 wk. if 100 w-hr. were required per chick per week?

CHAPTER 2

Percentage

Meaning of Per Cent

The phrase *per cent* is a contraction of the Latin *per centum,* which means *by the hundred* or *for each one hundred.* It is used in mathematics as having exactly the same meaning as the word *hundredths.* The phrase *5 per cent* means *5 hundredths* and may be written as .05, $\frac{5}{100}$, or $\frac{1}{20}$. The symbol for per cent is %. Per cents are convenient to use in making comparisons between quantities and rates, since one hundred is employed as a common basis.

Problems Involving Per Cents

In every problem containing a per cent three numbers are involved:

1. The *base,* of which a number of hundredths is taken.
2. The *rate per cent* (or simply *rate*), expressed in hundredths, by which the *base* is multiplied.
3. The *percentage,* which is the result of multiplying the *base* by the *rate.*

In the statement

$$8\% \text{ of } 365 = .08 \times 365 = 29.20,$$

.08 is called the *rate;* 365 is called the *base;* and 29.20 is called the *percentage.*

The word *percentage* is also used in a broader sense to mean the whole subject matter of per cents and computations with per cents.

A problem in percentage necessarily involves the finding of one of the three elements mentioned above from data

which gives the values of the other two elements. The three types of problems and the rule for solving each are given in the following:

1. PROBLEM: Finding a per cent of a number.
   Rule: Percentage = rate × base.
   Example: Find 12.5% of 368.
   $$12.5\% \text{ of } 368 = .125 \times 368 = 46.000$$
   or $$12.5\% \text{ of } 368 = \tfrac{1}{8} \times 368 = 46.$$

2. PROBLEM: Finding a number of which a per cent is known.
   Rule: Base = percentage ÷ rate.
   Example: \$155 is 40% of what amount?
   $$155 \div .40 = 387.50$$
   or $$155 \div \tfrac{2}{5} = 155 \times \tfrac{5}{2} = 387.50.$$
   \$155 is 40% of \$387.50.

3. PROBLEM: Finding what per cent one number is of another.
   Rule: Rate = percentage ÷ base.
   Example: 243 A. is what per cent of 682 A.?

243 ÷ 682 = .3563 approximately = 35.6% approximately.
243 A. is approximately 35.6% of 682 A.

Per Cent Equivalents

Every per cent is equivalent to a common fraction and may be replaced by it in making computations. Thus

$$37\tfrac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \tfrac{3}{8}.$$

The student will find it well worth while to memorize the following table of per cents and equivalent common fractions:

$25\% = \frac{1}{4}$	$20\% = \frac{1}{5}$	$10\% = \frac{1}{10}$	$12\frac{1}{2}\% = \frac{1}{8}$	$8\frac{1}{3}\% = \frac{1}{12}$
$50\% = \frac{1}{2}$	$40\% = \frac{2}{5}$	$30\% = \frac{3}{10}$	$37\frac{1}{2}\% = \frac{3}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$75\% = \frac{3}{4}$	$60\% = \frac{3}{5}$	$70\% = \frac{7}{10}$	$62\frac{1}{2}\% = \frac{5}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$100\% = 1$	$80\% = \frac{4}{5}$	$90\% = \frac{9}{10}$	$87\frac{1}{2}\% = \frac{7}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$

Per cents may be replaced by equivalent decimal numerals simply by moving the decimal point two places to the left (that is, dividing by 100) and leaving off the per cent symbol. For example,

$$8.75\% = .0875.$$

Problems

1. How many acres is 35% of 256 A.?

2. An inch is what per cent of a foot?

3. The most valuable constituent of milk is butterfat. How many pounds of butterfat are there in 15 gal. of milk testing 4.5%?

4. Calves lose about 40% of their live weight in dressing. Find approximately what the dressed weight of a 230-lb. calf should be.

5. A farmer decreased his wheat acreage of 80 A. by 15%. How many acres did he then have in wheat?

6. A farmer has 72 A. of land in cotton, and this is 45% of his entire farm. How many acres are there in his farm?

7. A man bought a piece of land for $4500 and sold it so as to gain 8% of what it cost him. How much did he gain and what was the selling price?

8. In conducting a germination test on seed corn a farmer found that 108 grains out of 150 germinated. What per cent germinated?

9. Of 350 eggs placed in an incubator 243 hatched. What per cent of the eggs hatched?

10. How much interest does a man pay on $300 borrowed for 8 mo. at $7\frac{1}{2}$% per annum?

11. What is the cash cost of 18 bales of barb wire at $3.90, subject to discounts of 10% and 2%? (It is understood that the second discount is on the remainder resulting from the first discount.)

12. How many gallons of milk testing 4.8% are required to produce 36 lb. of butterfat?

13. In making ice cream the volume of the unfrozen mixture may be increased 80% by whipping. How many gallons of mixture would be required to make 10 gal. of ice cream?

14. A farmer budded 265 pecan trees, one bud to a tree, and found that 171 buds lived. What per cent of the buds lived?

15. Nico-dust, which is used to kill lice on melons, is made by mixing "Black-Leaf 40" with lime. How much "Black-Leaf 40" would be used with 25 lb. of lime to make a dust testing 2% "Black-Leaf 40"?

16. The composition of average shelled corn is as follows: Protein, 10.1%; fat, 5.0%; ash, 1.5%; carbohydrates, 72.9%; and

water, 10.5%. How many pounds of each of these are there in a bushel of average corn?

17. A feeder gets a bill for $800 due in 60 da. with 2% off for cash. Would it pay him to borrow money from the bank at 8% per annum in order to take advantage of the cash discount?

18. The top $6\frac{2}{3}$ in. of soil, called the "top soil," is usually considered as weighing 2,000,000 lb. to the acre. If a sample of top soil from a certain field is found to be 0.15% nitrogen by weight, how many pounds of nitrogen per acre are there in the top soil?

19. Average milk is composed (by weight) of 3.5% fat, 3.5% protein, 4.6% lactose, 0.7% ash, and water. What per cent is water? How many pounds of water are there in a gallon of average milk?

20. A sample of soil weighing 189 g lost 39 g when thoroughly dried in an electric oven at 110° C. What was the per cent of moisture in the sample?

21. A farmer borrowed $250 on Jan. 15 at 8%. What amount, principal and interest, should he pay on Oct. 15 of the same year?

22. A man bought 12 head of cattle on Feb. 1 at $45 a head, promising to pay for them later with interest at 10%. If he paid the bill on the following Apr. 16, how much did he pay?

23. On a note for $375 at 8% a payment of $30 is credited annually at the close of each year from its date for 3 yr. How much is due at the end of the fourth year?

24. A farmer borrows $300 from a bank for 6 mo. at 8%. If the bank collects the interest in advance, the farmer actually has the use of what sum of money for the 6 mo.? What rate of interest is the farmer actually paying?

25. A butcher bought a prime 400-lb. steer at 6¢ per pound. The animal dressed 60%, and the average selling price of the meat per pound was $0.20. What was the total gross profit? What was the per cent of gross profit?

26. A farmer bought 300 baby chicks at 6¢ each. He lost 40, and the average cost of raising the rest to an average weight of 2 lb. was 14¢. If he sold them at 20¢ per pound, what was his profit? What was the per cent of profit?

27. If a horse is worth $160 at the age of 3 yr. but is worth only $24 at the age of 20 yr., what is the average annual depreciation? The average annual depreciation is what per cent of the value of the horse at the age of 3 yr.?

28. A formula developed by the Minnesota Agricultural Experiment Station for the approximate annual depreciation, d, of milch cows is

$$d = 0.015V_o + 0.16\tfrac{2}{3}(V_o - V_n),$$

where V_o is the number of dollars in the original value, and V_n is the number of dollars in the final "block" value. (The factor 0.015, or $1\frac{1}{2}\%$, is allowed for accidental death losses, and the $16\frac{2}{3}\%$ is based upon a 6-yr. milking life.) Counting the "block" value of a \$175 cow as \$30, find the approximate annual depreciation.

29. The life of a farm wagon is estimated as 20 yr. If the annual depreciation is assumed to be constant throughout the 20 yr., what is the rate of annual depreciation?

30. What is the expected life of a grain binder which depreciates approximately $8\frac{1}{3}\%$ of its original value each year?

31. Commercial fertilizers contain nitrogen, phosphoric acid, and potash in various proportions for different crops and soils. A fertilizer labeled 2-8-4 contains (by weight) 2% nitrogen, 8% phosphoric acid, and 4% potash. How many pounds of each of these are there in a ton of fertilizer so labeled?

32. Nitrate of soda is often used in fertilizers to supply the nitrogen. How many pounds of nitrate of soda testing 16% nitrogen would be required to furnish 40 lb. of nitrogen?

33. Phosphates may be used in fertilizers to provide the phosphoric acid needed. How much superphosphate yielding 20% phosphoric acid is required to provide 160 lb. of phosphoric acid?

34. Muriate of potash is a main source of potash in fertilizers. How many pounds of muriate of potash testing 50% potash are needed to supply 80 lb. of potash?

35. In Problems 32, 33, and 34 we have determined the number of pounds of essential materials, or plant food carriers (250 lb. of nitrate of soda, 800 lb. of superphosphate, and 160 lb. of muriate of potash), required to make the 2-8-4 fertilizer mentioned in Problem 31. Note that the sum of the weights of these carriers falls short of being a ton by 790 lb. Filler, usually in the form of sand or dry clay, is used to fill out the ton, thus making the fertilizer contain the proper per cents, 2-8-4, of nitrogen, phosphoric acid, and potash, respectively. Addition of filler also makes possible a more even distribution of the fertilizer. What per cent of the ton of fertilizer mentioned above is filler?

36. Practically the only plant foods contained in the fertilizer mentioned above are nitrogen, phosphoric acid, and potash. What per cent of the ton of fertilizer is plant food? What per cent is not plant food?

37. Compute the number of pounds of nitrogen, phosphoric acid, and potash in a ton of 4-8-4 fertilizer.

38. Find the number of pounds each of Chilean nitrate of soda testing 15% nitrogen, acid phosphate testing 16% phosphoric acid, and sulphate of potash testing 45% potash required to make the fertilizer mentioned in Problem 37.

39. How many pounds of filler are needed in the fertilizer mentioned in Problem 37? This filler makes up what per cent of the fertilizer?

40. What per cent of the fertilizer mentioned in Problem 37 is plant food? How many pounds of non-plant-food material are there in this ton of fertilizer?

41. Suppose that a farmer can buy the fertilizer mentioned in Problem 37 already mixed at $1.75 per hundredweight, and can buy the nitrate of soda at 3¢ per pound, the acid phosphate at 1¢ per pound, and the sulphate of potash at $2\frac{1}{2}$¢ per pound. Allowing for a 2% loss in mixing, and disregarding the cost of the labor of mixing, which is more economical for the farmer, to buy the ready-mixed fertilizer or to buy the materials and mix them?

42. How many pounds of ammonium sulphate are required to furnish the nitrogen in 1200 lb. of 3-8-4 fertilizer?

43. How many pounds of phosphate of lime are required to supply the phosphoric acid in 1800 lb. of 4-10-0 fertilizer?

Determine the per cent of nitrogen, per cent of phosphoric acid, and per cent of potash in each of the following fertilizer mixtures:

44. 1200 lb. of superphosphate (18%) and 800 lb. of cottonseed meal.

45. 1200 lb. of superphosphate (20%), 60 lb. of cottonseed meal, and 200 lb. of nitrate of soda.

46. 1080 lb. of acid phosphate, 800 lb. of cottonseed meal, and 120 lb. of muriate of potash.

47. 800 lb. of superphosphate (18%), 800 lb. of cottonseed meal, 200 lb. of nitrate of soda, and 200 lb. of muriate of potash.

48. How many pounds each of nitrate of soda testing 16%

nitrogen, superphosphate containing 18% phosphoric acid, and muriate of potash testing 50% potash are required for a ton of 5-10-5 fertilizer?

49. Barnyard manure contains about 0.6% nitrogen, 0.4% phosphoric acid, and 0.6% potash. If nitrogen is worth 15¢ per pound, phosphoric acid is worth 6¢ per pound, and potash is worth 5¢ per pound, what is the value of the plant food contained in a ton of manure?

50. What is the value of the plant food in the manure for a year from 125 head of sheep which weigh 80 lb. each and produce 34 lb. of manure per day per 1000 lb. of live weight if sheep manure tests 0.6% nitrogen, 0.3% phosphoric acid, and 0.55% potash and these plant foods are worth 12¢, 6¢, and 5¢ per pound, respectively?

51. The constituents of feeds, as indicated by most feed analyses, are (1) proteins, (2) ether extracts (consisting mainly of fats and oils), (3) nitrogen-free extract (consisting mainly of carbohydrates such as sugars and starches), (4) crude fiber (consisting chiefly of the cell walls and woody material of plants), (5) ash, which is the residue obtained when the feed is burned, and (6) water. Table VII, page 170, gives the average composition of certain feeding stuffs. If corn contains 10.4% crude protein, while only 6.4% of corn is digestible protein, what per cent of corn is indigestible protein? What per cent of the crude protein is indigestible?

52. By reference to Problem 51, determine how many pounds of digestible protein are contained in a bushel of shelled corn.

53. How many pounds of protein are there in this feed mixture for brood sows: Corn (shelled), 75 lb.; wheat gray shorts, 13 lb.; cottonseed meal, 7 lb.; and tankage, 5 lb.? What is the per cent of protein in this mixture?

Determine the per cent of protein in each of the following mixtures for dairy cattle:

54. 200 lb. of corn meal, 100 lb. of wheat bran, and 200 lb. of cottonseed meal.

55. 400 lb. of corn meal, 300 lb. of cottonseed meal, 100 lb. of ground oats, and 200 lb. of wheat bran.

56. 400 lb. of cottonseed, 200 lb. of alfalfa meal, and 100 lb. of wheat bran.

57. 100 lb. of ground oats, 200 lb. of dried beet pulp, 100 lb. of molasses, 100 lb. of wheat bran, and 300 lb. of cottonseed meal.

58. If the minimum weight upon which freight charges are made for a carload of hogs is 16,500 lb., the freight rate from College Station to Fort Worth is 23¢ per hundredweight, the commission for marketing a carload of hogs is $12, and a miscellaneous charge of $20 is made to cover feed, yardage, insurance, and so forth, what is the cost of marketing at Fort Worth a carload (of minimum weight) of hogs from College Station? If there is a shrinkage of $3\frac{1}{2}\%$ in the weight of the hogs due to shipping, and the hogs are sold at $10 per hundredweight, what is the net amount received by the shipper?

59. Find the cost of marketing a carload of cattle weighing 22,000 lb., assuming a freight rate of 26¢ per hundredweight, a commission charge of $7\frac{1}{2}$¢ per hundredweight, a miscellaneous charge covering yardage, insurance, and so forth, of 7¢ per hundredweight, and a feed charge of 2¢ per hundredweight. If there is a shrinkage of 5% in live weight, and the cattle are sold at $12.50 per hundredweight, what is the net amount received by the shipper for the cattle?

60. At large slaughtering plants, the dressed weight of an animal is taken to mean the weight of the carcass after 48 hr. of chilling. Actually, however, the warm carcass is weighed immediately after cleaning, and a reduction of 2% of the warm weight is allowed for loss in weight that will result from dripping and evaporation while chilling. If the live weight of a calf is 380 lb. and the warm carcass weighs 228 lb., the dressed weight is what per cent of the live weight?

61. The slope of land is an important factor affecting soil losses due to flow of water over the land. Slope, when expressed as a per cent, is usually taken to mean the number of feet of fall in elevation per 100 ft. of horizontal distance. If two points 100 ft. apart (horizontal distance) differ in elevation by 8.1 ft., what is the per cent of slope of land between the two points?

62. Two points 240 ft. apart (horizontal distance) differ in elevation by 15 ft. What is the slope, expressed as a per cent?

63. On a certain tract of land having a 3% slope the vertical interval between two terraces is $2\frac{1}{2}$ ft. How far apart horizontally are the terraces?

CHAPTER 3

Equations

Meaning of an Equation

In solving problems, it is sometimes convenient to use a letter to stand for a number whose value is to be found. After introducing a letter to represent an unknown number, a statement of equality may be written in which this letter is used just as the number it represents might be used if it were known. For example, consider the problem of finding the number of pounds of lint cotton that may be expected from 1260 lb. of clean seed cotton, assuming that the weight of the seed is twice the weight of the lint. We may let n stand for the number of pounds of lint. Then $2n$ stands for the number of pounds of seed, and we may write the relation

$$n + 2n = 1260,$$

or

$$3n = 1260,$$

from which

$$n = 420.$$

Therefore we conclude that the 1260 lb. of seed cotton would produce about 420 lb. of lint.

A statement of equality between two number expressions is called an *equation*. The two expressions on opposite sides of the "is equal to" symbol are called the *members*, or the *sides*, of the equation. In the above problem, the statement that $n + 2n = 1260$ is an equation and may be considered as involving the letter n in such a way as to make it serve

as the number it represents. Furthermore, if we substitute 420 for n in the equation above, we have

$$420 + 2(420) = 1260,$$

or

$$420 + 840 = 1260,$$

or

$$1260 = 1260.$$

Thus the two members of the equation become exactly the same when the proper value is substituted for n. This value of n (that is, 420) is said to *satisfy* the equation; and 420 is called a *root* of the equation.

Solving Equations

The process of finding the value of an unknown in an equation is called *solving the equation.* In solving an equation any changes may be made that do not destroy the equality of the members of the equation. The following operations are the main ones used in solving an equation, and each of these operations results in a new equation whose members are equal:

1. *Addition of the same number to each member.*
2. *Subtraction of the same number from each member.*
3. *Multiplication of each member by the same number.*
4. *Division of each member by the same number (not zero).*

Each of the equations obtained by performing one of the above operations imposes on the letter representing the unknown number the same value as is imposed by the original equation. By making successive changes in the form of an equation we try to arrive at an equation of the form $x =$ a certain number, which gives directly the information sought.

The student should recall that in multiplication or division of two numbers of like signs the result is positive, and that for two numbers of unlike signs the result is negative.

In performing operations with algebraic numbers, the student may find the table on the following page helpful.

OPERATIONS WITH ALGEBRAIC NUMBERS

Addition	Subtraction	Multiplication	Division
$6 + 2 = 8$ $6 + (-2) = 4$ $(-6) + 2 = -4$ $(-6) + (-2) = -8$	$6 - 2 = 4$ $6 - (-2) = 8$ $(-6) - 2 = -8$ $(-6) - (-2) = -4$	$(6)(2) = 12$ $(6)(-2) = -12$ $(-6)(2) = -12$ $(-6)(-2) = 12$	$6 \div 2 = 3$ $6 \div (-2) = -3$ $(-6) \div 2 = -3$ $(-6) \div (-2) = 3$
$6a + 2a = 8a$ $6a + (-2a) = 4a$ $(-6a) + (2a) = -4a$ $(-6a) + (-2a) = -8a$	$6a - 2a = 4a$ $6a - (-2a) = 8a$ $(-6a) - (2a) = -8a$ $(-6a) - (-2a) = -4a$	$(6a)(2a) = 12a^2$ $(6a)(-2a) = -12a^2$ $(-6a)(2a) = -12a^2$ $(-6a)(-2a) = 12a^2$	$6a \div 2a = 3$ $6a \div (-2a) = -3$ $(-6a) \div 2a = -3$ $(-6a) \div (-2a) = 3$
$(3a) + (2b) = 3a + 2b$	$(3a) - (2b) = 3a - 2b$	$(3a)(2b) = 6ab$	$(3a) \div (2b) = \dfrac{3a}{2b}$
$(3a - 2b) + (a - 5b)$ $= 3a - 2b + a - 5b$ $= 4a - 7b$	$(3a - 2b) - (a - 5b)$ $= 3a - 2b - a + 5b$ $= 2a + 3b$	$(3a + 2b)(a - 5b)$ $= 3a^2 - 15ab + 2ab - 10b^2$ $= 3a^2 - 13ab - 10b^2$	$(3a - 2b) \div (a - 5b) = \dfrac{3a - 2b}{a - 5b}$
$(5a^2 - 3ab + b^2)$ $+ (2a^2 - 4b^2) + (a^2 - 2ab)$ $\begin{array}{rrr} 5a^2 - 3ab & + b^2 \\ 2a^2 \quad\quad & - 4b^2 \\ a^2 - 2ab & \\ \hline 8a^2 - 5ab & - 3b^2 \end{array}$	$(5a^2 - 3ab - b^2)$ $- (2a^2 + 4ab - 3c)$ $\begin{array}{rr} 5a^2 - 3ab - b^2 & \\ 2a^2 + 4ab \quad\quad & - 3c \\ \hline 3a^2 - 7ab - b^2 & + 3c \end{array}$	$(a^2 - 3ab - b^2)(5a - 2b)$ $\begin{array}{rrrr} a^2 - & 3ab - & b^2 & \\ & 5a & + 2b & \\ \hline 5a^3 - & 15a^2b - & 5ab^2 & \\ + & 2a^2b - & 6ab^2 & - 2b^3 \\ \hline 5a^3 - & 13a^2b - & 11ab^2 & - 2b^3 \end{array}$	$(8a^3 - 27b^3) \div (2a - 3b)$ $\begin{array}{r} 4a^2 + 6ab + 9b^2 \\ 2a - 3b\overline{)8a^3 - 27b^3} \\ \underline{8a^3 - 12a^2b} \\ 12a^2b - 27b^3 \\ \underline{12a^2b - 18ab^2} \\ 18ab^2 - 27b^3 \\ \underline{18ab^2 - 27b^3} \end{array}$

Examples

1. Solve $\frac{1}{2}x = 14 - \frac{1}{5}x$ for x.

Multiply each member by 10 to clear of fractions:

$$10(\tfrac{1}{2}x) = 10(14 - \tfrac{1}{5}x),$$

or

$$5x = 140 - 2x.$$

Add $2x$ to each side:

$$7x = 140.$$

Divide each side by 7:

$$x = 20.$$

2. Solve $.07y + .05(1100 - y) = 59$ for y.

Perform the indicated multiplication:

$$.07y + 55.00 - .05y = 59.$$

Subtract 55 from each member:

$$.07y - .05y = 59 - 55.$$

Collect terms:

$$.02y = 4.$$

Divide each member by .02:

$$y = 200.$$

3. Solve $\dfrac{1}{n-2} = \dfrac{3}{n-3}$ for n.

Multiply each side by $(n - 2)(n - 3)$ to clear of fractions

$$(n - 2)(n - 3)\left(\frac{1}{n-2}\right) = (n - 2)(n - 3)\left(\frac{3}{n-3}\right),$$

or

$$n - 3 = 3(n - 2),$$

or

$$n - 3 = 3n - 6.$$

Add $3 - 3n$ to each member:

$$n - 3n = -6 + 3.$$

Collect terms:

$$-2n = -3.$$

Divide each side by (-2):

$$n = \frac{-3}{-2},$$

or

$$n = \tfrac{3}{2}.$$

Problems

Solve each of the following equations for the letter involved, indicating how each form of equation is obtained from the preceding form:

1. $3(4x - 3) + 15 = 2(x - 8).$

2. $16 + 4y - 4 = y + 12.$

3. $\dfrac{1}{5 - 2x} = \dfrac{2}{8x - 5}.$

4. $\dfrac{2w + 3}{11} + \dfrac{w - 1}{3} = 2.$

5. $\dfrac{17x - 5}{3} - \dfrac{10x + 2}{4} = \dfrac{5x + 7}{2} - 5.$

6. $0.04x = 0.1x + 2.4.$

7. $7m + 29 = 2m - 1.$

8. 50% of $(100 + n) = 47\frac{1}{2} + n.$

9. 2.5% of $600 + 30\%$ of $n = 3.5\%$ of $(600 + n)$. [This of course may be written as $.025(600) + .30n = .035(600 + n)$.]

10. $4\left(2y - \dfrac{1}{2}\right) - 6\left(\dfrac{y}{2} - \dfrac{1}{3}\right) = 2.$

11. $\dfrac{3x}{4} - \dfrac{7x}{12} = \dfrac{11x}{36} - \dfrac{8x}{9} + \dfrac{3}{2}.$

12. $\dfrac{2(x + 1)}{3} - \dfrac{3(x + 2)}{4} = \dfrac{x + 1}{6}.$

13. $x - \left(3x - \dfrac{2x - 5}{10}\right) = \dfrac{1}{6}(2x - 57) - \dfrac{5}{3}.$

14. $3(5 - 6x) - 5\{x - 5[1 - 3(x - 5)]\} = 23.$

15. $\dfrac{x + 1}{5} - \dfrac{x - 1}{2} = \dfrac{3 - x}{3}.$

16. $\dfrac{5}{x - 0.33} = \dfrac{5}{2}.$

17. $8(x - 4) + \frac{2}{3}(9 - 2x) = 14.$

18. $3\frac{1}{3}[28 - (\frac{1}{8}x + 24)] = 3\frac{1}{2}(2\frac{1}{3} + \frac{1}{4}x).$

Literal Equations

Many useful formulas are simply equations that involve several letters, each letter representing the measure of some

concrete quantity. Such equations are called *literal equations.* A literal equation may be solved for any one of the letters involved by considering all of the other letters as constants (numbers with fixed values), and using the methods employed in solving an equation containing only one letter.

Examples

1. The equation $I = Prt$ is the formula for finding the interest, I dollars, on a sum of money, P dollars, at rate r per year for t years.

To solve this equation for P, divide each side by rt:

$$P = \frac{I}{rt}.$$

To solve the equation for r, divide each side by Pt:

$$r = \frac{I}{Pt}.$$

2. $F = \frac{9}{5}C + 32$ is a formula expressing a temperature reading on the Fahrenheit scale in terms of a reading of the same temperature on the Centigrade scale. Solving this equation for C gives the formula for changing a Fahrenheit reading to a Centigrade reading.

$$F = \frac{9C}{5} + 32.$$

Multiply each member by 5:

$$5F = 9C + 160.$$

Subtract 160 from each side:

$$5F - 160 = 9C.$$

Divide each side by 9:

$$\frac{5F - 160}{9} = C.$$

Rewrite:

$$C = \frac{5F - 160}{9},$$

or

$$C = \frac{5}{9}(F - 32).$$

Problems

1. Solve $\frac{L_1}{L_2} = \frac{W_2}{W_1}$ for L_2.

2. Solve $C = 2\pi r$ for r.

3. Solve $A = P + Prt$ for P.

4. Solve $S = \frac{1}{2}n(a + L)$ for a.

5. Solve $L = a + (n - 1)d$ for n.

6. Solve $V = \frac{1}{3}\pi r^2 h$ for r.

7. Solve $A = 2\pi rh + \pi r^2$ for h.

8. Solve $\dfrac{F}{F_1} = \dfrac{2\pi L}{a}$ for L.

9. Solve $W = \dfrac{kbd^2}{L}$ for b.

10. Solve $S^2 = \dfrac{S^2}{4} + h^2$ for h.

11. Solve $\dfrac{a}{b} = \dfrac{c}{d}$ for a; for d.

12. Solve $A = \frac{1}{2}h(b_1 + b_2)$ for b_1. What is the value of b_1 when $A = 2400$, $h = 60$, and $b_2 = 50$?

13. Solve $I = Prt$ for r; for t. What is the value of r when $P = 1200$, $I = 210$, and $t = 3\frac{1}{2}$?

14. Solve $S = \dfrac{a + b + c}{2}$ for b. What is the value of b when $S = 30$, $a = 23.5$, and $c = 20.7$?

15. Solve $dr = DR$ for d.

16. Solve $ax - d = bx - c$ for x.

17. Solve $P = \dfrac{FH}{33{,}000}$ for F.

18. Solve $A = 2\pi(R - r)$ for r.

19. Solve $L = \pi(R + r) + 2d$ for d; for R.

20. Solve $V = \frac{1}{12}\pi h(2D^2 + d^2)$ for h.

21. Solve $\dfrac{P_2 - x}{x - P_1} = \dfrac{W_1}{W_2}$ for x.

22. Solve $I = \dfrac{En}{R + nr}$ for n.

23. Solve $A = td + b(s + n)$ for s.

24. The normal weight, W (in pounds), for a person of a cer-

tain height, h (in inches), is given approximately by the formula

$$W = 110 + \tfrac{11}{2}(h - 60).$$

Determine approximately the normal weight of a person of your height.

Applications of the Equation

Most of the problems that the student will meet in everyday life will be expressed in words and not in algebraic symbols. Before he can solve these problems, he must learn to translate English sentences into mathematical language. He will then find the equation a most important tool in solving problems. An example will illustrate this type of translation:

PROBLEM: How many pounds of cream testing 20% butterfat must be added to 80 lb. of milk testing 3% butterfat to give milk testing 4.5% butterfat?

Analysis: Let x = Number of pounds of cream to be added.

Then $.20x$ = Number of pounds of butterfat in this cream;

and $.03(80)$ = Number of pounds of butterfat in the 80 lb. of milk.

$80 + x$ = Number of pounds of final mixture,

and $.045(80 + x)$ = Number of pounds of butterfat in the final mixture.

Forming the equation: Now, is not the truth of the following statement evident?

"The number of pounds of butterfat in the unmixed quantities of milk and cream is equal to the number of pounds of butterfat in the final mixture."

This statement may be written

"The number of pounds of butterfat in the 80 lb. of milk plus the number of pounds of butterfat in the x lb. of cream equals the number of pounds of butterfat in the final mixture."

Translated into the language of mathematics, this statement becomes the equation

$$.03(80) + .20x = .045(80 + x).$$

When this equation is solved, our problem is solved. Solving, we obtain

$$2.40 + .20x = 3.600 + .045x.$$
$$.20x - .045x = 3.6 - 2.4.$$
$$.155x = 1.2.$$
$$x = 7.7,$$ approximately, and we conclude that approximately 7.7 lb. of cream are required in the mixture.

In every practical problem in which the equation is employed as a tool in effecting the solution, the student should be able to write out in words the statement of facts upon which his equation is based. In writing an equation one should always use abstract numbers rather than concrete numbers.

Problems

1. How many pounds of cream testing 25% butterfat must be added to 200 lb. of 3.5% milk to make a 5% milk? By 3.5% milk is meant milk that tests 3.5% butterfat.

2. How many pounds each of 3.5% milk and 6.2% milk are required to make 100 lb. of 4.5% milk?

3. How much skim milk must be mixed with 5.6% milk to make 100 lb. of 4% milk? Consider the skim milk as containing no butterfat.

4. If 10 lb. of 4.5% milk are mixed with 15 lb. of 5.2% milk, the mixture should test what per cent butterfat?

5. How much water must be added to 24 oz. of a 16% solution of salt to make a 2% solution?

6. How many gallons of water must be added to 20 gal. of a winter spray of lime-sulphur testing 15% lime-sulphur, by volume, to make a summer spray which tests 5% lime-sulphur?

7. Counting 24 tablespoons to a pint, determine how many tablespoons of disinfectant should be added to a gallon of water to make a 10% solution.

8. The value of 38 coins consisting of dimes and quarters is $5.30. Find the number of each kind of coin.

9. A part of $800 is invested at 3% per annum and the remainder at 4%. The yearly income from the two investments is $30. Find the amount of each investment.

10. Sound travels at about the rate of 1100 ft. per second. How far from an observer is a gun whose flash is observed $1\frac{1}{2}$ sec. before the report is heard?

11. Two automobiles start from points 225 mi. apart toward each other on a straight road, one traveling at an average rate of 40 mi. per hour and the other at 35 mi. per hour. They should meet after how many hours?

12. The sum of the angles of a triangle is 180°. Find the angles of a triangle in which one angle is twice the size of the smallest angle and the largest angle is three times the size of the smallest angle.

13. It is estimated that a new set of spark plugs, costing $4.80, will increase the mileage of a certain car from 14 mi. to 15 mi. per gallon of gasoline. If gasoline costs 17¢ per gallon and the increased mileage is maintained sufficiently long, how many miles must the car be driven before the saving on gasoline pays for the new plugs?

14. A piece of wire 4 ft. long is to be cut into two pieces so that the longer piece will be 6 in. more than twice as long as the shorter piece. Find the length of each piece.

15. A dealer pays $24 for a suit of clothes. At what price should he mark the suit so as to allow a discount of $33\frac{1}{3}\%$ of the marked price and still make a profit of 25% of the cost?

16. A dealer marked a pair of shoes at $7.50 but sold them at a discount of 20% and still made a profit of 25% of the cost. What did the shoes cost the dealer? If the shoes had been sold at the marked price, what would have been the per cent of profit?

17. How many pounds of cottonseed meal testing 41.9% crude protein should be added to 100 lb. of feed testing 16.3% crude protein to produce a feed containing 23% crude protein?

18. How many pounds of acid phosphate should be added to 800 lb. of cottonseed meal to form a fertilizer containing 10% phosphoric acid? Determine the per cent each of nitrogen and potash in the mixture.

19. How many pounds each of wood ashes and cottonseed meal should be used to form 1000 lb. of fertilizer containing 3% potash?

20. A stock solution for an oil spray consists of 2 gal. of oil weighing 7 lb. 8 oz. per gallon, 2 lb. of soap, and 1 gal. of water. Determine the per cent of oil in this stock solution.

21. How much water should be added to 1 gal. of the stock solution mentioned in Problem 20 to make a winter spray testing 3% oil for use against the San Jose scale? (Disregard volume occupied by soap in solution.)

22. How much water should be added to 1 gal. of the stock solution mentioned in Problem 20 to make a 1% oil spray for spraying citrus trees to control the red orange scale. (Disregard volume occupied by soap in solution.)

23. "Black Leaf 40" is a brand of nicotine sulphate which weighs about 10 lb. to the gallon and tests 40% nicotine. If a spray used against plant lice consists of 1 pt. of "Black Leaf 40" to 100 gal. of water, what per cent "Black Leaf 40" does it contain? What per cent nicotine does it contain?

24. For use as a dip against sheep scab a solution testing 0.07 of 1% nicotine is formed by adding "Black Leaf 40" to water. How many pounds of "Black Leaf 40" should be added to 100 gal. of water to make a dip of this strength?

Systems of Linear Equations

Many problems require that the values of two or more unknown quantities be determined. In solving such problems it may be convenient to introduce several letters to represent the unknowns. It then becomes necessary to form as many equations as there are letters involved and to solve this group of equations. A group of equations under consideration at the same time is called a *system of simultaneous equations.*

Solving a system of two linear equations in two unknowns is usually brought about by combining, by addition or subtraction, the two members of one equation with the corresponding members of the other in such a way as to eliminate one of the unknowns, thereby resulting in a third equation that contains only one unknown.

Example

Solve for x and y: $\begin{cases} (1) & 4x - 3y = 6. \\ (2) & x + 5y = 13. \end{cases}$

To eliminate x, multiply each side of Equation (2) by 4. We then have

$$\begin{cases} (1) & 4x - 3y = 6. \\ (2) & 4x + 20y = 52. \end{cases}$$

Subtract: $\qquad -23y = -46.$

Divide by (-23): $\qquad y = 2.$

The value of x may be found by substituting 2 for y in either one of the original equations and solving the resulting equation in x. Thus with $y = 2$, Equation (2) becomes

$$x + 5(2) = 13,$$

or $\qquad x + 10 = 13,$

from which $\qquad x = 3.$

The value of x might have been determined by multiplying each side of Equation (1) by 5 and each side of Equation (2) by 3 and adding, thereby eliminating y.

$$\begin{aligned} 20x - 15y &= 30. \\ 3x + 15y &= 39. \\ \hline 23x \qquad &= 69. \\ x &= 3. \end{aligned}$$

The solution of the system of equations consists of the pair of numbers 3 and 2. This means that each of the equations is a true statement if $x = 3$ and $y = 2$.

Problems

1. Solve for x and y: $\begin{cases} x + 2y = 7. \\ 5x - 2y = 11. \end{cases}$

2. Solve for h and k: $\begin{cases} 10h - k = -3. \\ 12h + 12k = 102. \end{cases}$

3. Solve for x and y: $\begin{cases} \dfrac{3x - 20}{2} = \dfrac{2x + 5y.}{3} \\ 10 = x - y. \end{cases}$

4. Solve for m and n: $\begin{cases} .04m + .75n = 10. \\ .8m - 1.25n = 5. \end{cases}$

5. Solve for x and y: $\begin{cases} x + y = 200. \\ .4x + .5y = 92. \end{cases}$

6. How many pounds each of 3% milk and 5% milk should a farmer mix to make 100 lb. of milk testing 4.5%?

7. An airplane flew 400 mi. against the wind in 5 hr. and made the return trip in 3 hr. and 20 min. Assuming that the difference in time was due solely to the speed of the wind, find the speed of the airplane in still air and how fast the wind was blowing.

8. How many pounds of bone meal testing 3% nitrogen and 24% phosphate and dried blood testing 13% nitrogen are needed for an acre of wheat ground which is strong in potash but which requires 12 lb. of nitrogen and 20 lb. of phosphate per acre?

9. How many pounds of acid phosphate containing 16% phosphoric acid must be added to 800 lb. of a 2-6-4 fertilizer to increase the per cent of phosphoric acid to 8%?

10. A feed dealer wishes to make up 1000 lb. of corn chops, which he can sell at $1.60 per hundredweight, by mixing two grades, one retailing regularly at $1.50 and the other at $1.75 per hundredweight. How many pounds of each grade should he use?

11. How many pounds each of two kinds of grain, one worth $1.50 per hundredweight and the other worth $3 per hundredweight, should be used to form 1000 lb. of a mixture worth $2 per hundredweight?

12. How many pounds each of two kinds of feed, one worth 2¢ a pound and one worth 4¢ a pound, are required to make 200 lb. worth $2\frac{1}{2}$¢ a pound?

13. How many pounds each of cottonseed meal and acid phosphate should be used to make 1000 lb. of a fertilizer testing 4% nitrogen and 7.5% phosphoric acid? (The 1000 lb. may include some filler.)

14. How many pounds each of acid phosphate, cottonseed meal, and muriate of potash should be used to make 1000 lb. of a 4-7-4 fertilizer? (The 1000 lb. may include some filler.)

CHAPTER 4

Lengths, Areas, and Volumes

Measurements as Approximations

In actual practice, measurements of length, weight, volume, time, and so forth, are, in general, only approximations. For example, if we say that a certain piece of rope is 30 ft. long, we really mean that it is approximately 30 ft. long. It may be 29 ft. 10 in., or perhaps 29 ft. $10\frac{1}{2}$ in. expresses more nearly the length of the rope. The fact is that it is usually impossible to state with certainty the exact length of the piece of rope. Such factors as inaccuracy of the measuring instruments, inability of a person to manipulate and read the instruments accurately, and the variable condition of the rope—its temperature, moisture content, degree of tension, and so forth—all tend to make the measurement inexact.

However, in most cases sufficient accuracy of measurement can be had for all ordinary purposes. It may be that strict adherence to 30 ft. as the length of the piece of rope implies a greater degree of accuracy than is needed; that depends upon how the rope and the measure of its length are to be used.

In computations involving numbers which represent practical measurements the results should not indicate a higher degree of accuracy than is warranted by the precision of the measurements made. Particularly in performing a division in which the dividend is an approximation, the student should not continue a quotient to more and more decimal places by annexing zeros to the dividend.

Units of Measure

Measuring a quantity of any kind consists in comparing that quantity with a certain other quantity of the same kind, known as the *unit of measure.* For instance, to say that a board is 10 ft. long means that its length is 10 times as great as a certain other length, called a *foot,* which is taken as a unit for measuring length. In a similar manner, to say that an object weighs $86\frac{1}{2}$ lb. means that the weight of the object is $86\frac{1}{2}$ times as great as a certain other weight, called a *pound,* which is taken as a unit for measuring weight.

The student is probably familiar with most of the units of measure in common use. However, certain units of measure may require a brief discussion.

Angles

If two straight line segments, OA and OB, proceed from the same point, they are said to form an angle. (See Fig. 1.) The point O is called the *vertex* of the angle, and the line segments OA and OB are called the *sides* of the angle. The angle may be denoted by the single letter O or by the symbol $\angle AOB$ (read "angle AOB").

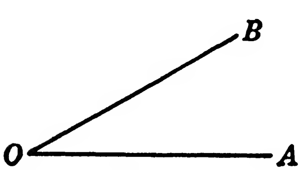

Fig. 1.

The size of an angle does not depend upon the lengths of its sides. In fact, it is convenient to consider that the size of an angle is determined by the amount of turning that one line segment, say OA, must do about the point O in order to become coincident with the other line segment. If the line segment OA is rotated completely about the point O (so that it comes back to its original position), then OA is said to generate an angle of *one revolution.* The *revolution,* then, is a unit of angular measure. The unit of angular measure most often used is the *degree,* which may be defined as $\frac{1}{360}$ of a revolution. The degree is divided into 60 smaller units, called *minutes,* and the minute is further

divided into 60 units called *seconds*. These relationships are shown in the following table:

1 revolution = 360 degrees, written 360°.
1 degree = 60 minutes, written 60′.
1 minute = 60 seconds, written 60″.

An angle of 90° is called a *right angle*.

An angle of 180° is called a *straight angle*.

Angles less than 90° are said to be *acute*.

Angles greater than 90° and less than 180° are said to be *obtuse*.

If two intersecting line segments form a right angle, each line segment is said to be *perpendicular* to the other.

Areas

A square whose side is 1 in. long contains a *square inch;* a 1-ft. square contains a *square foot;* a 1-mi. square contains a *square mile;* and so forth. The number of square inches,

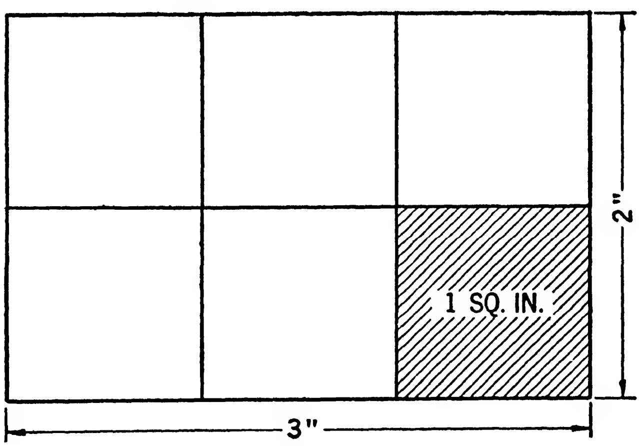

Fig. 2.

square feet, or square units of any kind contained within a figure is spoken of as the *area* of the figure. For example, if a rectangle 3 in. long and 2 in. wide is divided into 1-in. squares, as shown in Fig. 2, it is seen to have an area of 6

sq. in. This area may be expressed in square feet by noting that

$$1 \text{ sq. ft.} = 144 \text{ sq. in.},$$
$$1 \text{ sq. in.} = \tfrac{1}{144} \text{ sq. ft.},$$

and

$$\begin{aligned} 6 \text{ sq. in.} &= 6\left(\tfrac{1}{144}\right) \text{ sq. ft.} \\ &= \tfrac{6}{144} \text{ sq. ft.} \\ &= \tfrac{1}{24} \text{ sq. ft.} \end{aligned}$$

Volumes

A cube whose edge is 1 in. long contains a *cubic inch;* a 1-ft. cube contains a *cubic foot;* and so forth. The number of cubic units of any kind contained in a figure is spoken of

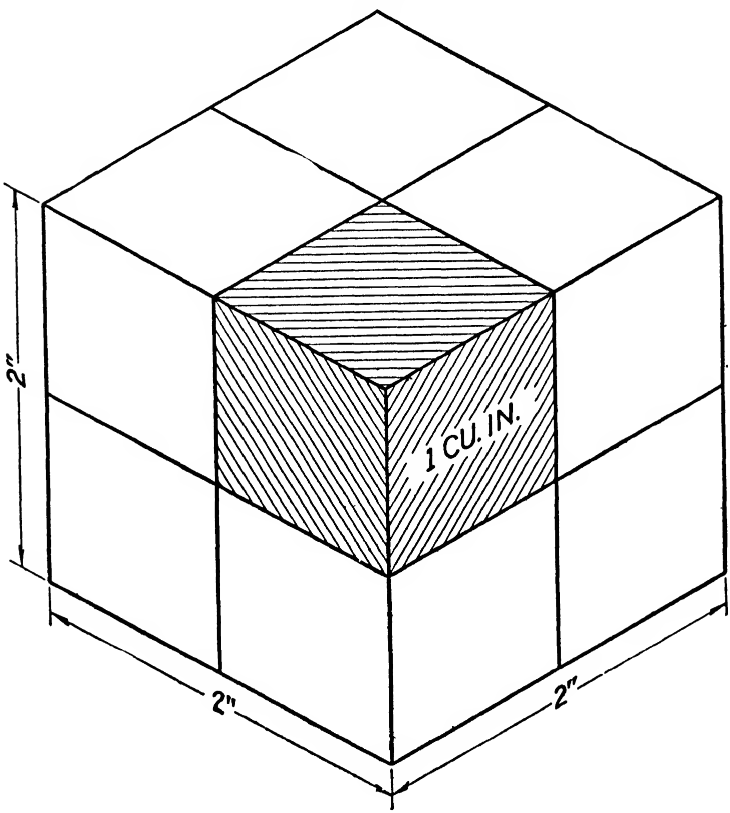

Fig. 3.

as the *volume* of the figure. For example, if a box-shaped figure 2 in. long, 2 in. wide, and 2 in. high is divided into 1-in. cubes, as illustrated in Fig. 3, it is seen to have a volume of 8 cu. in. Note that a 2-in. cube is really eight times as large as a 1-in. cube.

On the next few pages an outline of the plane and solid figures most often involved in practical measurements is given. The student should find the formulas useful in computing distances, areas, and volumes.

Plane Figures

1. Triangle (figure bounded by three straight line segments).

(a) General Triangle (triangle of any shape).

b = base.
h = altitude.

$$\text{Perimeter} = a + b + c.$$

$$\begin{aligned}\text{Area} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}ab \sin C \\ &= \tfrac{1}{2}\, ac \sin B \\ &= \tfrac{1}{2}bc \sin A \\ &= \sqrt{s(s-a)(s-b)(s-c)}, \\ &\quad \text{where } s = \tfrac{1}{2}(a+b+c).\end{aligned}$$

$$\begin{aligned}\angle A + \angle B + \angle C &= 180° \\ &= \pi \text{ radians.}\end{aligned}$$

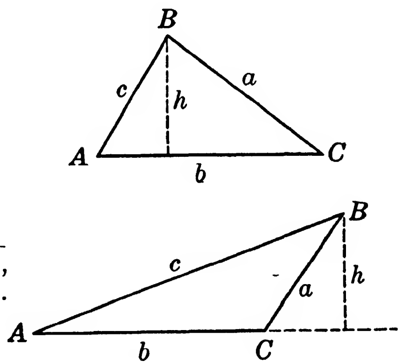

Fig. 4.

(b) Isosceles Triangle (triangle which has two sides equal).

$$a = c.$$
$$AD = DC.$$
$$\angle A = \angle C.$$

$$c^2 = h^2 + \left(\frac{b}{2}\right)^2.$$

$$\begin{aligned}h &= \sqrt{c^2 - \frac{b^2}{4}} \\ &= \tfrac{1}{2}\sqrt{4c^2 - b^2}.\end{aligned}$$

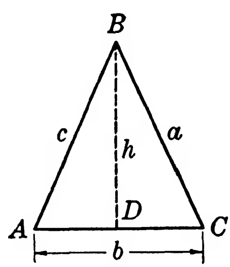

Fig. 5.

(c) Equilateral Triangle (triangle whose sides are equal).

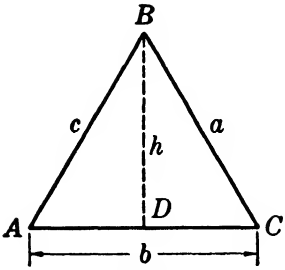

Fig. 6.

Formulas under (a) apply.
$a = b = c.$

Perimeter $= 3a = 3b = 3c.$

$\angle A = \angle B = \angle C = 60°.$

$$h = \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2}b = \frac{\sqrt{3}}{2}c.$$

(d) Right Triangle (triangle in which one angle is 90°).

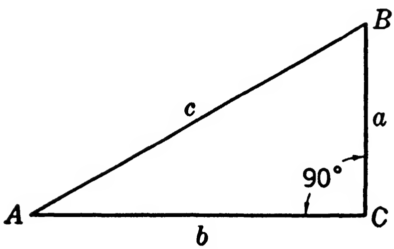

Fig. 7.

Formulas under (a) apply.

Area $= \frac{1}{2}ba.$

$c^2 = a^2 + b^2.$

(A more complete discussion of the right triangle is given in the next chapter.)

2. Quadrilateral (plane figure bounded by four straight lines).

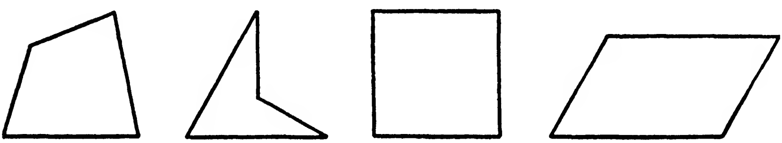

Fig. 8.

Perimeter = sum of sides.
Area: There is no simple general formula for the area. In the following sections formulas are given for the areas of certain types of quadrilaterals.

(a) Trapezium (quadrilateral in which no two sides are parallel).

Perimeter = sum of sides.
Area = sum of areas of two triangles.

$A = \frac{1}{2}bh_1 + \frac{1}{2}bh_2.$

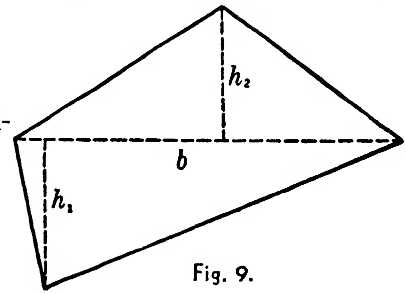

Fig. 9.

(b) Trapezoid (quadrilateral having only two sides parallel).

Perimeter = sum of sides.
Area = $\frac{1}{2}$ (upper base + lower base) × altitude.

$A = \frac{1}{2}(b_1 + b_2)h.$

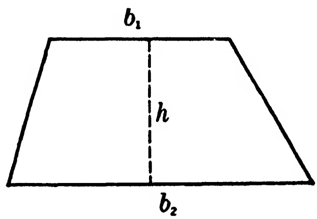

Fig. 10.

(c) Parallelogram (quadrilateral having opposite sides parallel and equal).

Perimeter = (2 × longer side) + (2 × shorter side).
Area = base × altitude.
$A = bh.$

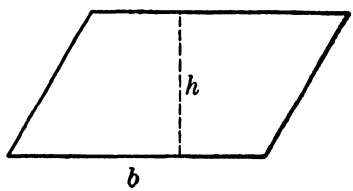

Fig. 11.

(d) Rectangle (parallelogram whose angles are right angles).

Perimeter = (2 × base) + (2 × altitude).
Area = base × altitude.
$A = bh.$
Diagonal = $\sqrt{(\text{base})^2 + (\text{altitude})^2}$
$d = \sqrt{b^2 + h^2}.$

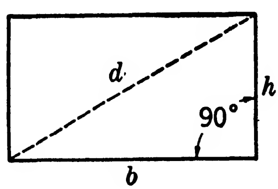

Fig. 12.

(e) Square (rectangle whose sides are equal).

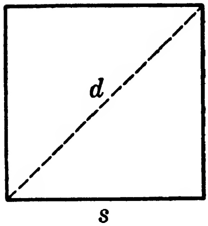

Fig. 13.

Perimeter $= 4 \times$ side.
Area $=$ (side)^2.
$A = s^2$.
Diagonal $= \sqrt{2} \times$ side.
$d = \sqrt{2}s$.

3. Circle (plane figure bounded by a curved line, all points of which are equidistant from a fixed point called the center).

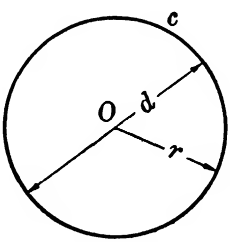

Fig. 14.

$O =$ center.
$\pi = 3.1416$, approximately
$= \frac{22}{7}$, approximately.
$r =$ radius.
$d =$ diameter.
$c =$ circumference.
$A =$ area.

$d = 2r$.
$c = 2\pi r = \pi d$.
$A = \pi r^2$.

4. Sector of a Circle (portion of circle bounded by two radii and their intercepted arc).

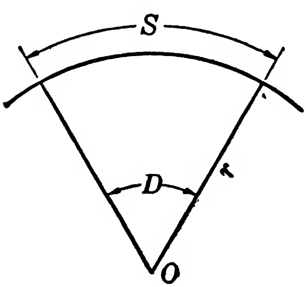

Fig. 15.

$r =$ radius.
$S =$ arc.
$D =$ degrees in central angle.
$A =$ area.

$$S = \frac{\pi r D}{180^\circ}.$$
$$A = \tfrac{1}{2}rS = \frac{\pi r^2 D}{360^\circ}.$$

5. The Trapezoidal Rule for Approximate Area. If a figure is bounded on one side by an irregular curve and on the other sides by straight lines, as illustrated in Fig. 16, the approximate area of the figure can usually be expressed satisfactorily as the sum of the areas of a number of trapezoids constructed as follows. The base line, AB, is divided into n segments of equal length d, and at each point of division of AB, as well as at A and B, perpendiculars are erected. The points of intersection of these perpendiculars with the curve CD are then joined by straight line segments, so that n trapezoids are formed. If the lengths of the $(n + 1)$ perpendicular segments extending from AB to points on CD are denoted by $h_0, h_1, h_2, \ldots, h_n$, the approximate area of the figure $ABCD$ is given by the formula

$$\text{Area} = d(\tfrac{1}{2}h_0 + h_1 + h_2 + \ldots + h_{n-1} + \tfrac{1}{2}h_n).$$

For any particular choice of n, the student may readily verify that this formula gives the sum of the areas of the n trapezoids. In an application of the formula the choice of n depends upon the degree of accuracy desired—usually, the larger n is, the better is the approximation.

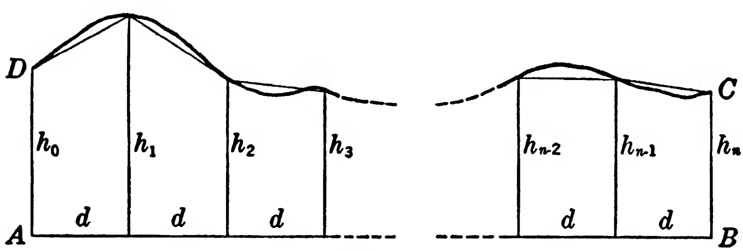

Fig. 16.

6. Simpson's Rule. Another method of approximating the area of a figure of the type illustrated by Fig. 16 is based upon the assumption that the figure is divided into an *even* number of vertical strips of equal width and bounded above by arcs of parabolas. This method leads to Simpson's rule for approximate area:

$$\text{Area} = \frac{d}{3}(h_0 + 4h_1 + 2h_2 + 4h_3 + 2h_4 + \ldots + 4h_{n-1} + h_n).$$

Solids

1. Rectangular Solid (solid bounded by six rectangles).

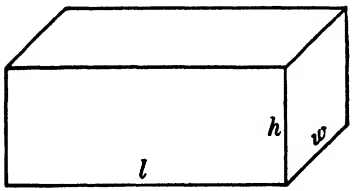

Fig. 17.

Volume = length $\times$ width $\times$ depth.

$V = l \times w \times h.$

Area of surface = sum of the areas of the six rectangular faces.

$A = 2lw + 2lh + 2wh.$

2. Cube (rectangular solid whose edges are all equal).

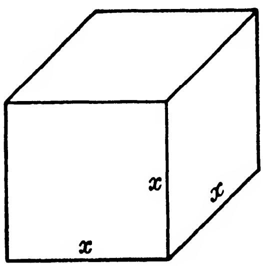

Fig. 18.

Volume = $(\text{edge})^3$.

$V = x^3.$

Area of surface = $6 \times (\text{edge})^2$.

$A = 6x^2.$

3. Right Prism (solid bounded by two polygons, called the bases, which are congruent and lie in parallel planes, and a number of rectangular lateral faces perpendicular to the bases, there being a lateral face passing through each pair of corresponding sides of the two polygons).

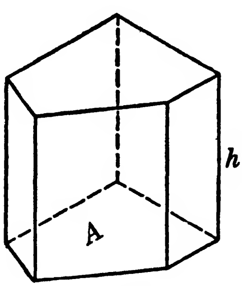

Fig. 19.

h = altitude.

A = area of base.

Lateral area = sum of areas of lateral faces.

Total area = lateral area + area of bases.

Volume = Ah.

(Note that this formula for volume applies to right cylinders also and that a rectangular solid is a special type of prism.)

4. Right Circular Cylinder (solid generated by revolving a rectangle about one of its sides).

r = radius.
h = altitude.

Volume $= \pi r^2 h$.
Lateral area $= 2\pi rh$.
Total surface area $= 2\pi rh + 2\pi r^2$.

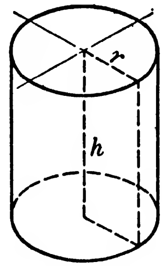

Fig. 20.

5. Right Circular Cone (solid generated by revolving a right triangle about one of its legs).

r = radius of base.
h = altitude.
s = slant height.

Volume $= \frac{1}{3}\pi r^2 h$.
Lateral area $= \pi rs$.
Total area $= \pi rs + \pi r^2$.

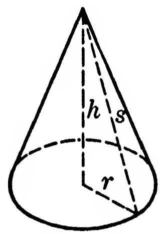

Fig. 21.

6. Frustum of Right Circular Cone (portion of a right circular cone included between the base and a cross section parallel to the base).

h = altitude.
A_1 = area of lower base of radius r_1.
A_2 = area of upper base of radius r_2.
s = slant height.
C_1 = circumference of lower base.
C_2 = circumference of upper base.

$$\text{Volume} = \frac{h}{3}(A_1 + A_2 + \sqrt{A_1 A_2})$$

$$= \frac{\pi h}{3}(r_1^2 + r_1 r_2 + r_2^2).$$

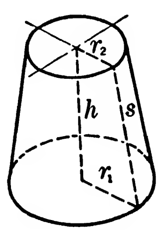

Fig. 22.

$$\text{Lateral area} = s\,\frac{(C_1 + C_2)}{2}.$$

7. Regular Pyramid (solid bounded by three or more congruent triangles having a common vertex and a regular polygon whose sides are the bases of the triangles).

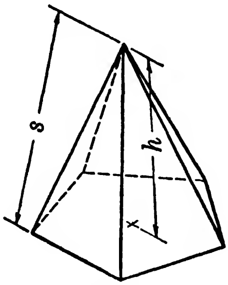

Fig. 23.

A = area of base.
h = altitude.
s = slant height.
P = perimeter of base.

Volume $= \frac{1}{3}Ah$.

Lateral area $= \dfrac{Ps}{2}$.

8. Frustum of Regular Pyramid (portion of a regular pyramid included between the base and a cross section parallel to the base).

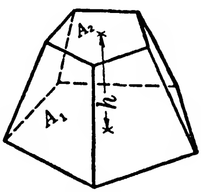

Fig. 24.

h = altitude.
A_1 = area of lower base.
A_2 = area of upper base.
V = volume.

$$V = \frac{h}{3}(A_1 + A_2 + \sqrt{A_1A_2}).$$

9. Sphere (solid bounded by a curved surface all points of which are equidistant from a fixed point called the center).

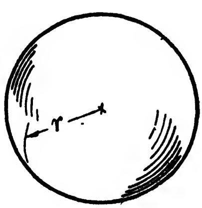

Fig. 25.

r = radius.

Volume $= \frac{4}{3}\pi r^3$.
Area $= 4\pi r^2$.

10. Barrel.

D = middle diameter.
d = end diameter.
h = height.

Volume $= \frac{1}{12}\pi h(2D^2 + d^2)$, approximately.

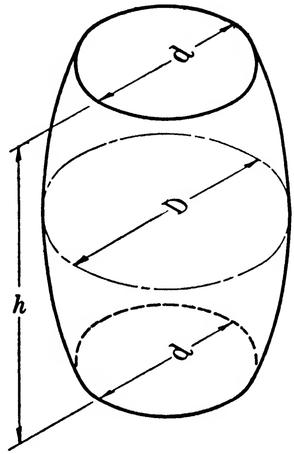

Fig. 26.

The formula given here for the approximate volume of a barrel is a special application of the more general formula

$$V = \frac{h}{6}(B_1 + B_2 + 4A),$$

in which h is the altitude of a solid, measured between parallel bases (each or either of which may reduce to a point), B_1 and B_2 are the areas of the parallel bases, and A is the area of the cross section parallel to the bases and midway between them. This formula, known as the *prismoidal formula*, is worth remembering, since, except for the barrel, it gives the exact volume of every solid mentioned above and gives satisfactory approximations for the volumes of many other solids.

11. Torus, or Anchor Ring (solid obtained by revolving a circle about an axis in the plane of the circle and not intersecting the circle).

a = radius of circle.
b = distance from center of circle to axis of revolution.

Surface area $= 4\pi^2 ab$.
Volume $= 2\pi^2 a^2 b$.

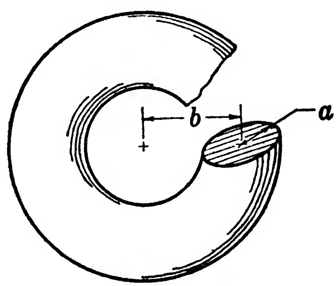

Fig. 27.

Problems

1. How many square yards of canvas are required for the walls and ceiling of a room 16 ft. by 18 ft. if the walls are 9 ft.

high and a deduction of one-fifth the area of the walls is made on account of windows and doors?

2. How many acres are there in a rectangular field 1580 ft. long and 675 ft. wide?

3. How many square feet of sheet iron are required to make the bottom and wall of a cylindrical cistern 6 ft. in diameter and 8 ft. high?

4. What must be the length of a binding rod for a cylindrical silo 14 ft. in diameter if 1 ft. is allowed for overlapping?

5. Find the number of acres in a field that is practically in the form of a trapezoid whose bases are 140 yd. and 105 yd. and whose altitude is 70 yd.

6. If a room is heated by 210 ft. of steam pipe 2 in. in diameter, what is the area of the radiating surface?

7. What is the diameter of a tree around which a 16-ft. rope will just reach?

8. How many squares of roofing are required for a gable roof each side of which is a rectangle about 18 ft. by 53 ft.? How much will the roofing cost at $5.20 per square if it cannot be bought in lots of less than $\frac{1}{2}$ square?

9. About how many gallons of paint should be required for two coats on the walls of a barn 32 ft. by 48 ft. if the walls are 12 ft. high and no allowance is made for openings?

10. A silo is 16 ft. in diameter and 36 ft. high. About how many gallons of paint would be required to give the wall of the silo two coats?

11. Each of two portions of a hip roof is a trapezoid with bases of 40 ft. and 25 ft. and an altitude of 18 ft., and each of two portions is an isosceles triangle with a base of 20 ft. and an altitude of 20 ft. How many 4-in. shingles should be required for the roof?

12. How many revolutions per second is a 30-in. (outside diameter) automobile wheel making when the automobile is traveling 40 mi. per hour?

13. A 3-ft. walk is constructed around a circular pool whose diameter is 80 ft. At $1.80 per square yard what is the cost of the walk?

14. Find the total pressure on a piston 6 in. in diameter when the pressure guage registers 140 lb. per square inch.

15. How many square yards of canvas are there in a conical tent whose base is 14 ft. in diameter and whose slant height is 12 ft.?

16. Allowing 2 in. for rolling at the edges, how many square feet of tin are required in making 120 ft. of gutter whose cross section is a semicircle $3\frac{1}{2}$ in. in diameter? (Use $\frac{22}{7}$ for π.)

17. Counting three pickets 2 in. by 1 in. by 4 ft. for each linear foot of fence, determine how many gallons of paint are required to give 100 ft. of picket fence two coats.

18-22. Find the number of acres in the fields represented by Figs. 28–32. The figures are not to be considered as drawn to scale.

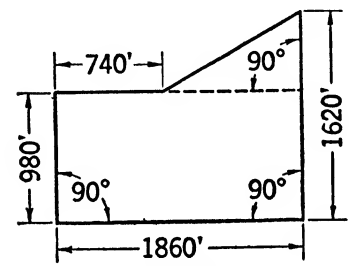

Fig. 28.

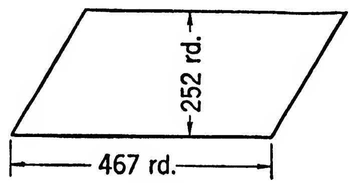

Fig. 30.

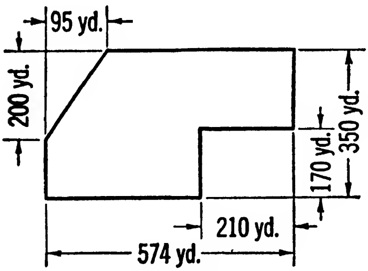

Fig. 29.

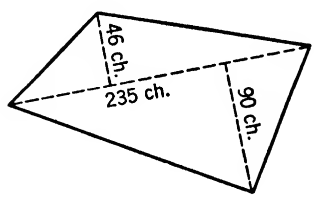

Fig. 31.

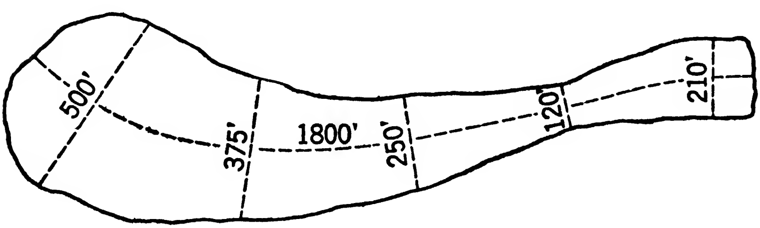

Fig. 32.

23-27. Assume that Figs. 33–37 are each drawn to scale in representing a certain field. By measuring the side of the drawing upon which is written the length of the corresponding side of the field, determine the scale used. Then find approximately the lengths of other field lines needed in computing the area, dotting in auxiliary lines. Finally compute the approximate area of the field.

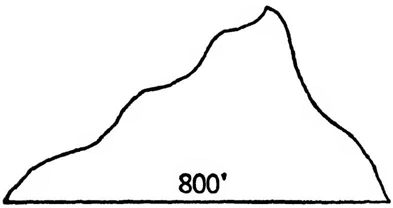

Fig. 33.

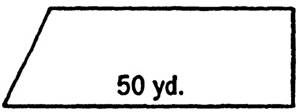

Fig. 34.

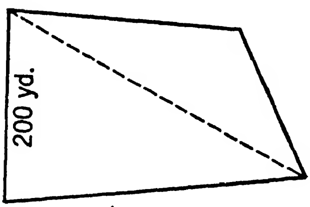

Fig. 35.

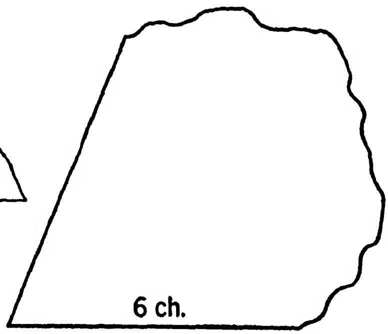

Fig. 36.

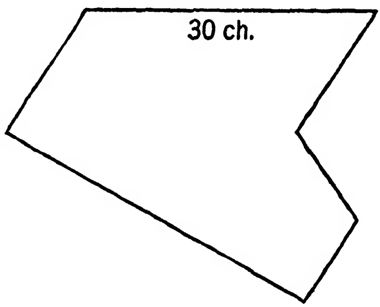

Fig. 37.

28. In Fig. 16, page 41, fill in the breaks in the curved line CD and the base line AB and insert another altitude, h_4, halfway between the one labeled h_3 and the one labeled h_{n-2}. Then consider the figure $ABCD$ as having been drawn to scale in representing a certain field whose side corresponding to AB is 100 ch. long, and find approximately the number of acres in the field.

29. Fig. 38 is a drawing made from a portion of an aerial photographic map used in measuring acreages in connection with the Government's farm program. The portion here shown pictures all of a certain farm. The scale is 1 in. to 660 ft. In practice, areas are usually obtained from such pictures by use of a

planimeter. However, fairly satisfactory approximations may be computed from carefully made measurements of lengths on the field pictures. Determine approximately the number of acres in each of the fields lettered.

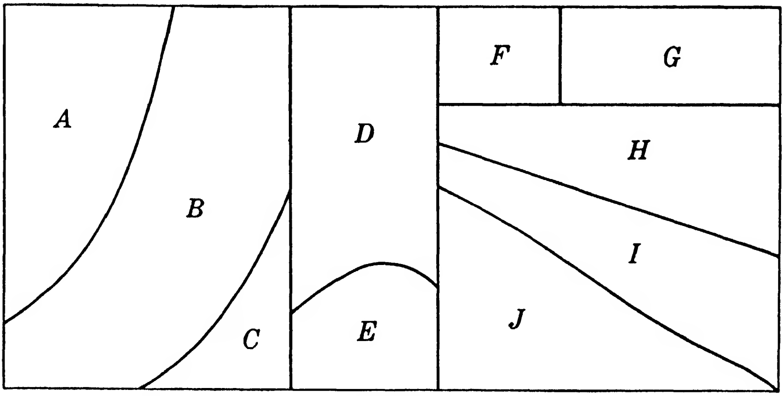

Fig. 38.

30. A wheat bin is 8 ft. long, 6 ft. wide, and 4 ft. deep. How many bushels of wheat does it contain if it is three-fourths full?

31. About how many cubic yards of earth are removed in making a pond which is almost in the form of a hemisphere, the greatest depth being 9 ft.? How many gallons of water should the pond hold when it is full?

32. About how many tons of silage are there in a cylindrical silo whose inside diameter is 12 ft. and whose height is 30 ft. if the silage lacks 5 ft. of reaching the top?

33. How many gallons of water does a cylindrical tank 5 ft. in diameter and 7 ft. high hold when it is full? How much does this volume of water weigh?

34. At $37\frac{1}{2}$¢ per cubic yard what is the cost of excavation for a cellar 24 ft. by 18 ft. by 9 ft.?

35. About how many tons of hay are there in a stack 10 ft. high, 30 ft. long, 16 ft. wide at the base, and 8 ft. wide at the top? (Volume = area of cross section $\times$ length.)

36. The interior of a freight car is 8 ft. wide, 34 ft. long, and 7 ft. high. If it is filled with wheat to a depth of 5 ft., what is the approximate weight of the wheat?

37. How many cords of wood are there in a pile 8 ft. wide, 8 ft. high, and 40 ft. long?

38. About how many bricks are needed in building a wall 42 ft. long, $1\frac{1}{2}$ ft. thick, and $6\frac{1}{4}$ ft. high? At \$9 per thousand what should be the cost of the bricks?

39. About how many bricks are required for the wall of an underground cylindrical cistern 7 ft. in diameter and 15 ft. deep if the wall is 4 in. thick? (Assume that the difference in outside and inside circumference is taken care of by wedgelike vertical mortar joints.)

40. If a haystack is considered as a right circular cylinder 10 ft. in diameter and 15 ft. high, how many tons of hay does it contain?

41. An ordinary farm-wagon box is 10 ft. long, 3 ft. wide, and 2 ft. deep. About how many bushels of shelled corn will it hold? About how many bushels of dry sound corn in the era will it hold?

42. Find the capacity in gallons of a vessel 20 in. deep whose bottom and top diameters are 7 in. and 14 in., respectively. (Apply the formula for the volume of the frustum of a cone.)

43. How many cubic feet of concrete are there in the wall of a cylindrical silo whose inside diameter is 12 ft. and whose wall is 8 in. thick and 30 ft. high?

44. A hog trough of triangular cross section measures 16 in. across the top, $8\frac{1}{2}$ in. in depth, and 4 ft. in length (inside measurement). How many gallons of water will it hold? (Volume = area of cross section $\times$ length.)

45. A compartment with a capacity of 1 cu. yd. is to be formed in an ordinary farm-wagon box by a partition placed across the wagon. How long should the compartment be? (See Problem 41.)

46. How high must a bin be made to hold 360 bu. of wheat if it is to be built on floor space 10 ft. long and 5 ft. wide?

47. What should be the length of a field 30 ch. wide if the field is to contain 150 A.?

48. At what height above the bottom of an ordinary wagon box should a mark be made to have the wagon contain a cubic yard when filled to that mark? (See Problem 41.)

49. In order to determine approximately the number of cubic feet per bushel to be allowed in measuring a certain lot of ear corn, a man shells a representative sample of 1 cu. ft. of the ear

corn and finds that the shelled corn weighs 22 lb. This result indicates how many cubic feet to the bushel?

50. How many bushels of ear corn of the kind mentioned in the preceding problem should there be in a truckload if the truck bed is 12 ft. long, 5 ft. wide, and 4 ft. deep?

51. Water is being pumped into a cylindrical tank 14 ft. in diameter at a rate such that, if no water is being drawn off, the water level rises 1 ft. in 20 min. How many gallons of water per hour are being delivered by the pump?

52. The cross section of a certain stream is practically a rectangle 20 ft. long and 8 ft. deep. It is observed that a piece of cork floats downstream a distance of 200 ft. in 3 min. If the mean velocity of the water is taken as 0.8 of the surface velocity, about how many gallons of water per hour pass a given point in the stream?

53. If a certain pipe 2 in. in diameter discharges 6 gal. of water per minute, determine the velocity of the water in feet per second at the point of issuance from the pipe.

54. How many gallons of water per minute will be discharged by a pipe $1\frac{1}{2}$ in. in diameter if the water has a velocity of $1\frac{1}{4}$ ft. per second?

55. How many feet deep should ear corn be piled in a crib 15 ft. long, 12 ft. wide, and 10 ft. deep to have the crib contain 250 bu. of corn?

56. Determine the dimensions of a rectangular piece of tin that can be bent to form the wall of a cylindrical can 4 in. in diameter and $4\frac{1}{2}$ in. tall, allowing $\frac{1}{4}$ in. for overlapping at the joint.

57. How many square feet of sheet iron are required for the wall of a cylindrical tank 7 ft. in diameter and 8 ft. high if a 9-in. strip 8 ft. long is allowed for overlapping?

58. How many feet deep should wheat be placed in a crib 12 ft. long, 10 ft. wide, and 8 ft. high so that the crib will contain 480 bu. of wheat?

59. How many feet deep should water be in a cylindrical tank 7 ft. in diameter and 7 ft. tall to have the tank contain 1000 gal. of water?

60. Find in feet per minute the speed of a belt being driven by a pulley 7 in. in diameter and making 1650 r.p.m. (revolutions per minute), assuming that belt slippage is negligible.

61. A square inch is equivalent to how many square centimeters?

62. A cubic inch is equivalent to how many cubic centimeters?

63. An ounce (avoirdupois) is equivalent to how many grams?

64. A pound is equivalent to how many grams? A pound is equivalent to how many kilograms?

65. A liter is equivalent to how many cubic inches?

66. The specific gravity of a substance is defined as the ratio of the weight of any volume of the substance to the weight of an equal volume of water. Since 1 cc of water at 4° C. weighs 1 g, the specific gravity of a substance may be considered as the number of grams in the weight of 1 cc of the substance. If alcohol has a specific gravity of 0.82, how much do 150 cc of alcohol weigh?

67. If a 10% solution of menthol in alcohol has a specific gravity of 0.854, what volume (in cubic centimeters) should 128 g of the solution have?

68. Glycerin has a specific gravity of 1.25. How much do 150 cc of glycerin weigh? How many cubic centimeters of glycerin weigh 75 g?

69. How many grams of silver nitrate should be dissolved in water to form 150 g of solution testing 10% silver nitrate?

70. How many grams of sodium chloride should be dissolved in water to form 1500 g of a 16% solution of the salt?

71. How many grams of sodium chloride should be added to 1500 cc of water to form a 16% solution of the salt?

72. How many grams each of menthol and alcohol are required to make 150 g of a 10% solution of menthol in alcohol? If the alcohol has a specific gravity of 0.82, how many cubic centimeters are required?

73. In how much water should $\frac{1}{2}$ lb. of salt be dissolved to form a 15% solution of salt?

74. How many grams of 5% soda solution can be made from 250 g of soda?

75. If 65 gr. of salt are dissolved in 400 gr. of water, what per cent of the resulting solution is salt?

76. If 300 cc of water are combined with 500 cc of nitric acid having a specific gravity of 1.4, what per cent (by weight) of the resulting product is nitric acid?

77. Suppose that a pharmacist is directed to provide exactly 100 cc of a 15% (by weight) solution of a certain salt in water and that the specific gravity of the desired solution is not known. How can he fill the prescription? (*Suggestion:* The pharmacist may use 100 cc of water, which weighs 100 g, to make more than 100 cc of the 15% solution and then use only 100 cc of the solution in filling the prescription.)

78. A physician orders 240 cc of a 3% (by weight) solution of mercuric chloride in water. How can the order be filled?

CHAPTER 5

Ratio and Proportion

Ratio

The ratio of one number to another is simply the common fraction obtained by dividing the one by the other. For example, the ratio of 3 to 5 is $\frac{3}{5}$ (sometimes written 3:5). The ratio of 8 in. to 12 in. is

$$\frac{8 \text{ in.}}{12 \text{ in.}}, \text{ or } \frac{8}{12}, \text{ or } \frac{2}{3}.$$

Ratios are abstract numbers. For a ratio between two concrete numbers to have meaning, the concrete numbers must be of the same kind; that is, they must be expressed in terms of the same unit.

Proportion

A proportion is simply an equation between two ratios. Hence a proportion involving an unknown is solved by the methods used in solving a fractional equation.

Example

Find the value of x if 3 is to x as 6 is to 35.
This statement may be written

$$3 : x = 6 : 35.$$

The preferred form, however, is

$$\frac{3}{x} = \frac{6}{35}.$$

Multiply each side by $35x$:

$$105 = 6x.$$

Divide each side by 6:

$$x = \tfrac{105}{6} = \tfrac{35}{2} = 17\tfrac{1}{2}.$$

If four numbers a, b, c, and d, none of which is zero, satisfy the proportion

$$(1) \quad \frac{a}{b} = \frac{c}{d},$$

it follows that they satisfy also the proportions

$$(2) \quad \frac{a}{c} = \frac{b}{d}$$

and

$$(3) \quad \frac{b}{a} = \frac{d}{c}.$$

The student should verify these statements.

Suppose that for each applicable value of a variable x another variable y has a corresponding value and that y_1 and y_2 are values of y that correspond, respectively, to the values x_1 and x_2 of x. Then to say that *y is proportional to x* means that y_1 is to y_2 as x_1 is to x_2; that is,

$$\frac{y_1}{y_2} = \frac{x_1}{x_2}.$$

To say that *y is inversely proportional to x* means that y_1 is to y_2 as x_2 is to x_1; that is,

$$\frac{y_1}{y_2} = \frac{x_2}{x_1}.$$

Stated below are a few principles whose applications may involve proportions.

1. The distance traveled by a body moving at a constant rate is proportional to the time of traveling. If a man can walk d_1 mi. in t_1 hr. and maintains this rate for t_2 hr., the number, d_2, of miles walked in t_2 hr. is given by the proportion

$$\frac{d_2}{d_1} = \frac{t_2}{t_1}.$$

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2. The cost of a number of articles bought at a fixed price per article is proportional to the number of articles bought. If N_1 acres of land are bought for P_1 dollars, then the cost, P_2 dollars, of N_2 acres bought at the same price per acre is given by the proportion

$$\frac{P_2}{P_1} = \frac{N_2}{N_1}.$$

3. Two geometric figures are considered similar if lengths measured on one of them are proportional to corresponding lengths measured on the other.
4. The areas of two similar plane figures are proportional to the squares of corresponding length measurements.
5. The volumes of two similar solids are proportional to the cubes of corresponding length measurements.

The proportions accompanying Figs. 39–42 illustrate how principles 3, 4, and 5 apply. In the proportions stated, the V's represent volumes, the A's represent areas, and the other letters represent lengths.

Similar Triangles

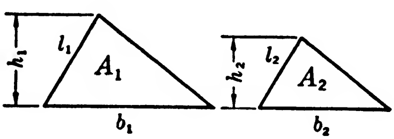


Fig. 39.

(1) $\frac{l_1}{l_2} = \frac{b_1}{b_2} = \frac{h_1}{h_2}.$

(2) $\frac{A_1}{A_2} = \frac{b_1^2}{b_2^2} = \frac{h_1^2}{h_2^2} = \frac{l_1^2}{l_2^2}.$

Circles

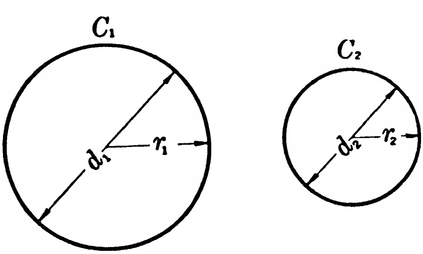


Fig. 40.

(1) $\frac{C_1}{C_2} = \frac{r_1}{r_2} = \frac{d_1}{d_2}.$

(2) $\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \frac{C_1^2}{C_2^2} = \frac{d_1^2}{d_2^2}.$

Similar Rectangular Solids

(1) $\dfrac{l_1}{l_2} = \dfrac{w_1}{w_2} = \dfrac{h_1}{h_2}.$

(2) $\dfrac{A_1}{A_2} = \dfrac{w_1^2}{w_2^2} = \dfrac{h_1^2}{h_2^2}.$

(3) $\dfrac{V_1}{V_2} = \dfrac{l_1^3}{l_2^3} = \dfrac{w_1^3}{w_2^3} = \dfrac{h_1^3}{h_2^3}.$

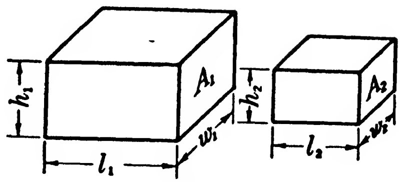


Fig. 41.

Spheres

(1) $\dfrac{C_1}{C_2} = \dfrac{r_1}{r_2}.$

(2) $\dfrac{A_1}{A_2} = \dfrac{r_1^2}{r_2^2}.$

(3) $\dfrac{V_1}{V_2} = \dfrac{r_1^3}{r_2^3}.$

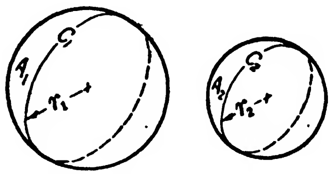


Fig. 42.

6. If a lever is balanced on a fulcrum and supports two weights or other forces applied at points on opposite sides of the fulcrum, the weights are inversely proportional to their distances from the fulcrum (provided that the weight of the lever itself can be neglected). The three forms of the lever are illustrated in Fig. 43, in which f denotes the position of the fulcrum, W_1 and W_2 are the forces, and L_1 and L_2 are distances usually referred to as the lever arms of W_1 and W_2, respectively. In each case we have the proportion

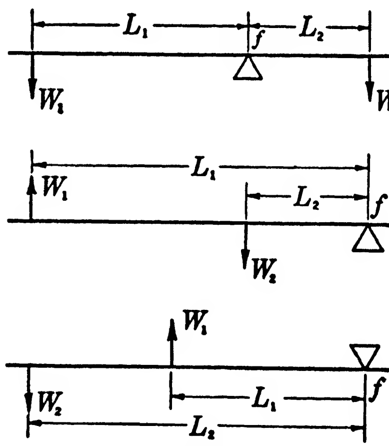


Fig. 43.

$$\frac{W_1}{W_2} = \frac{L_2}{L_1}.$$

Example

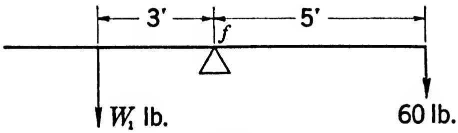


Fig. 44.

A uniform beam 10 ft. long is supported at the middle and carries a weight of 60 lb. at one end. What force must be applied at a distance of 2 ft. from the other end to balance the beam?

$$\frac{W_1 \text{ lb.}}{60 \text{ lb.}} = \frac{5 \text{ ft.}}{3 \text{ ft.}}.$$

$$\frac{W_1}{60} = \frac{5}{3}.$$

$$3W_1 = 300.$$

$$W_1 = 100.$$

7. Two pulleys connected by a belt revolve at rates that are inversely proportional to the diameters of the pulleys. In Fig. 45 d_1 and d_2 denote the diameters of two pulleys,

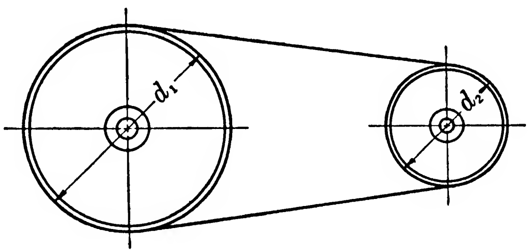


Fig. 45.

and R_1 and R_2 denote, respectively, their numbers of revolutions per unit of time, so that we have the proportion

$$\frac{R_1}{R_2} = \frac{d_2}{d_1}.$$

8. Two meshing gears revolve at rates that are inversely proportional to the numbers of teeth on the gears. If a gear having N_1 teeth and making R_1 r.p.m. meshes with a gear having N_2 teeth, the number, R_2, of revolutions per minute made by the latter gear is given by the proportion

$$\frac{R_2}{R_1} = \frac{N_1}{N_2}.$$

9. If (by use of some sort of machine such as a block and tackle arrangement of pulleys) a force of F_1 lb. acts through a distance of d_1 ft. to move a load of F_2 lb. through a distance of d_2 ft., then, neglecting friction involved in the machine, we have the proportion

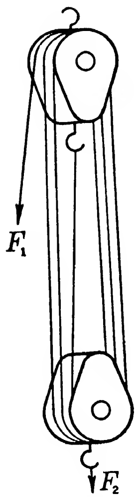


Fig. 46.

$$\frac{F_1}{F_2} = \frac{d_2}{d_1}.$$

For example, if forces of friction were negligible, the arrangement of six pulleys indicated in Fig. 46 could be employed to lift a load by a force equal to one sixth of the weight of the load. Note that the applied force F_1 would act through six times as great a distance as would the resisting force F_2. Thus, in this case

$$d_1 = 6d_2,$$

and

$$\frac{F_1}{F_2} = \frac{d_2}{6d_2}.$$

That is,

$$\frac{F_1}{F_2} = \frac{1}{6},$$

or

$$F_1 = \tfrac{1}{6}F_2.$$

In forming proportions, a brief comparison of each ratio with 1 often helps the student to avoid writing an obviously false proportion in which one ratio is greater than 1 and the other is less than 1.

Problems

1. A certain triangle whose shortest side is 20 ft. is similar to a triangle whose sides are 14 in., 10 in., and 8 in. Find the lengths of the other sides of the large triangle.

2. A square whose side is 60 ch. is how many times as large (in area) as a square whose side is 30 ch.?

3. At the instant when the shadow of a 6-ft. pole is 15 ft. long, the shadow of a certain tree is 90 ft. long. How high is the tree?

4. To find approximately the height of a barn a farmer sights from a point 3 ft. above the ground directly over the top of a 10-ft. pole to the highest point of the barn; he then measures his distance from the pole and his distance from the barn and finds them to be 8 ft. and 40 ft., respectively. How high is the barn?

5. If oranges 2 in. in diameter without rind are worth 16¢ per dozen, what should be the value of oranges of the same quality 3 in. in diameter?

6. If 2 cu. ft. of lime and 5 cu. ft. of sand are used in making 6 cu. ft. of mortar, how much of each is needed to make 63 cu. ft. of mortar?

7. Two machine pulleys, 20 in. and 12 in. in diameter, respectively, are connected by a belt. Find the speed of the smaller pulley when the larger is making 140 r.p.m. (The ratio of their speeds is equal to the inverse ratio of their diameters.)

8. Two boys, one weighing 70 lb. and the other weighing 90 lb., sit at opposite ends of a 16-ft. see-saw board. If the board is balanced and its weight is neglected, how far is the fulcrum from the end occupied by the larger boy?

9. The specific gravity of a substance is defined as the ratio of the weight of a given volume of that substance to the weight of an equal volume of water. If water weighs 62.5 lb. per cubic foot and cement weighs 100 lb. per cubic foot, what is the specific gravity of cement?

10. If an iron ball 2 in. in diameter weighs 1.1 lb., how much should a ball 6 in. in diameter weigh?

11. Neglecting the weight of the lever, find the force a man would need to apply at one end of an 8-ft. crowbar to lift a weight of 390 lb. situated at the other end, the fulcrum being $1\frac{1}{2}$ ft. from the weight.

12. What is the ratio of the capacities of two cube-shaped boxes if the edge of the larger is twice the edge of the smaller?

13. What is the ratio of the areas of two circles if the radius of the larger is twice the radius of the smaller? What is the ratio of their circumferences?

14. An 8-ft. lever whose fulcrum is at one end supports a load of 200 lb., concentrated at a point 2 ft. from the fulcrum. Neglecting the weight of the lever, find the lifting force required at the other end of the lever.

15. What force is necessary to lift a weight of 840 lb. by an arrangement of six pulleys in a block and tackle? What force is required if a four-pulley arrangement is used?

16. If a beef animal that is fed a daily ration of 2 lb. of cottonseed meal, 10 lb. of corn, and 12 lb. of prairie hay gains 1.9 lb. daily, how much of each of these feeds should be required to produce 100 lb. of live weight? How many pounds of this feed mixture is required per pound of gain?

17. It is estimated that for hogs 4 lb. of balanced feed is required per pound of gain in live weight. If hogs are fed a mixture consisting of 82% shelled corn, 5% wheat gray shorts, 9% cottonseed meal, and 4% tankage, how much of each of these feeds would be required to increase a hog's weight 100 lb.?

18. In rations that contain silage, as much as half the silage may be replaced by cottonseed hulls at the rate of 2 lb. of hulls to each 5 lb. of silage. How many pounds of hulls would be required to make this replacement in a daily ration that includes 60 lb. of silage?

19. In a ton of 3-8-4 fertilizer what is the ratio of the weight of the phosphoric acid present to the combined weight of the other plant foods present?

20. If a cow is to be fed a certain daily ration at the rate of 1 lb. of feed for each 3 lb. of milk produced per day, how much of this feed should be given a cow that averages $4\frac{1}{2}$ gal. of 5% milk a day?

21. A motor-driven pulley 8 in. in diameter is connected by belt to a service pulley 10 in. in diameter. When the 8-in. pulley is making 120 r.p.m., how fast is the service pulley revolving?

22. What size service pulley should replace the one mentioned in problem 21 if it is desirable to have the service pulley make only 80 r.p.m.?

23. The ratio between the diameters of two circles is $\frac{1}{3}$. What is the ratio between their areas? Between their circumferences?

24. If a steel ball 4 in. in diameter weighs 9 lb., what should be the weight of a ball of similar material that measures 6 in. in diameter?

25. An edge of a certain cube-shaped box is three times as long as the edge of another cube-shaped box. The larger box will hold how many times as much as the smaller box?

26. Neglecting the weight of the lever, find the force required at a point 6 in. from one end of an 8-ft. lever to balance a weight of 650 lb. placed at the other end, the fulcrum being 1 ft. from the weight.

27. Neglecting the weight of the lever, find the distance from one end of an 8-ft. lever a force of 30 lb. should be applied to balance a weight of 60 lb. placed at the other end, which is $2\frac{1}{2}$ ft. from the fulcrum.

28. Two horses are pulling a wagon, the total pull on the load being 300 lb. If the hitch for one of the horses is at one end of a 48-in. evener (or doubletree) and the hitch for the other horse is located 2 in. from the other end, determine the pull exerted by each of the horses. Assume that the doubletree is pivoted at its center and that the horses are kept abreast.

29. If the pulls exerted on a load by two mules should be proportional to the weights of the mules, devise a plan for properly hitching two mules, one weighing 1000 lb. and the other weighing 1200 lb., to a 48-in. doubletree, which is pivoted at its center.

30. A tractor pulley that is 12 in. in diameter and turns 950 r.p.m. is to drive an irrigation pump at 1750 r.p.m. About what size pulley should be used on the pump?

31. A 3-h.p. gas engine with a 6-in. pulley turning 500 r.p.m. is to drive a pulley of a pump jack at 250 r.p.m. What size pulley is needed on the pump jack?

32. One of two meshing spur gears on adjoining shafts has 24 teeth and the other has 10 teeth. How fast is the 10-tooth gear turning when the 24-tooth gear is making 1200 r.p.m.?

33. At what speed will a 6-in. pulley turn if it is belted to a $9\frac{1}{2}$-in. tractor pulley that makes 1000 r.p.m.?

34. A 10-in. motor-driven pulley making 800 r.p.m. is belted to an 8-in. pulley that is on the same shaft with another 8-in. pulley, which is joined by a belt to a 12-in. machine pulley. Find the speed of the 12-in. pulley.

35. A motor making 3650 r.p.m. has its $1\frac{3}{4}$-in. driving pulley connected with a 4-in. pulley on a lathe. Find the speed of the lathe pulley.

CHAPTER 6

The Right Triangle and Trigonometry

Theorem of Pythagoras

Certain properties of the right triangle are involved in many problems connected with construction work. By far the most important property of the right triangle is that known as the Theorem of Pythagoras, which states that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Applied to triangle ACB in Fig. 47, this theorem means that $c^2 = a^2 + b^2$, where a, b, and c are the number of units in the lengths of the sides. The student may recall the proof of this theorem from his study of geometry. A few of the applications of this property are suggested in problems which follow.

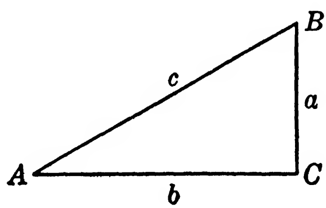


Fig. 47.

Finding the Square Root of a Number

In solving problems relating to the right triangle, the student needs to know how to find the square root of a number. The method is indicated in the following examples.

Examples

1. Find the square root of 2209.

First, group the digits of the number by two's from right to left, thus:

$$\overline{22}\quad\overline{09}$$

(In some cases, this procedure leaves only one digit in the group farthest to the left.)

Second, determine the greatest perfect square in the group of digits farthest to the left. In this case it is 16. Write down the square root of this number (in this case, 4) as the first figure of the square root. Subtract the 16 from the 22 and bring down the next group of digits in line with the remainder.

$$\begin{array}{rr|l}
\overline{22} & \overline{09} & \underline{4} \\
16 & & \\
\overline{6} & 09 &
\end{array}$$

Third, multiply the part of the root found so far (in this case, 4) by 20 and place the product opposite the 6 09 as a trial divisor.

$$\begin{array}{r|rr|l}
 & \overline{22} & \overline{09} & \underline{4} \\
 & 16 & & \\
\underline{80} & \overline{6} & \overline{09} &
\end{array}$$

Fourth, divide 609 by 80 and write the quotient, 7, to the right of the 4 already found. Also add a 7 to the 80 and multiply the 87 by 7.

$$\begin{array}{r|rr|l}
 & \overline{22} & \overline{09} & \underline{47} \\
 & 16 & & \\
\underline{87} & \overline{6} & \overline{09} & \\
 & \underline{6} & \underline{09} &
\end{array}$$

The square root of 2209 is seen to be exactly 47.

2. Find the square root of 151.29.

Beginning at the decimal point, group digits by two's in both directions, thus:

$$\overline{1}\ \ \overline{51}\ .\ \overline{29}$$

From here on proceed as in the other example, pointing off in the square root half as many decimal places as are in the number.

$$\begin{array}{r|rrl|l}
 & \overline{1} & \overline{51} & .\ \overline{29} & \underline{12.3} \\
 & 1 & & & \\
\underline{22} & & \overline{51} & & \\
 & & 44 & & \\
\underline{243} & & \overline{7} & \overline{29} & \\
 & & 7 & \underline{29} &
\end{array}$$

The square root of 151.29 is 12.3. If a number is not a perfect square, its square root can be found to any number of decimal places by annexing zeros and continuing the root extraction process.

Problems

1. There are 4840 sq. yd. in 1 A. The square root of 4840, then, is the number of yards in the length of the side of a square whose area is 1 A. Find it.

2. Find the number of rods in the side of a square containing 1 A. by extracting a square root.

3. Find the number of feet in the side of a square containing 1 A.

4. The rectangular frame for a gate is 3 ft. wide and 4 ft. high. Find the length of a brace to be placed diagonally across the gate.

5. The foot of a 45-ft. ladder is 27 ft. from the wall of a building against which the top rests. How high does the ladder reach on the wall?

6. How many feet is the radius of a circle whose area is 1 A.?

7. How long a guy wire is needed to reach from the top of a 40-ft. pole to a point 30 ft. from the foot of the pole if 1 ft. is allowed at each end for fastening?

8. A baseball diamond is 90 ft. square. How far is it from second base to home plate?

Pitch of Roof

Fig. 48 indicates the meaning of certain terms and measurements used in connection with roof construction. The *pitch* of a roof is defined as the ratio of the rise of the roof to the span, or it may be expressed as the ratio of the rise to twice the run. *Pitch* is sometimes incorrectly used as synonymous with *slope*. The *slope* of a roof is the ratio of the rise to the run.

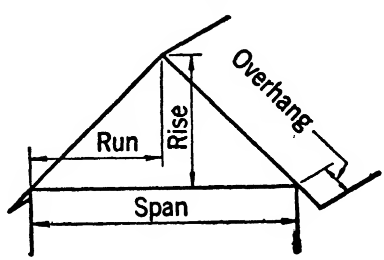


Fig. 48.

A common rafter of a gable roof (not counting the over-

hang) is seen to be the hypotenuse of a right triangle whose sides are the run and the rise of the roof. Therefore the length of such a rafter can be found by the principle of the theorem of Pythagoras if the other dimensions are known.

In actual practice a carpenter usually "steps off" the length of a rafter by placing a steel square on the side of the rafter material in such a way that 12 in. on one arm of the square corresponds to the run and a portion (depending upon the pitch) of the other arm represents the rise, and then moving the square along the piece of lumber by taking as many "steps" as there are feet in the run. By this method the markings for the top and bottom cuts on the rafter are also readily obtained.

Problems

1. How long a rafter is required for a building 36 ft. wide if the rise is 12 ft. and there is no overhang?

2. What length rafter would be cut for a roof whose pitch is $\frac{1}{4}$ if the roof has a span of 28 ft. and an overhang of 18 in.?

The Trigonometric Ratios

In the right triangle ACB in Fig. 49 the side AB is called the *hypotenuse* of the triangle; and, with reference to angle A, the side AC is called the *adjacent side* and the side CB is called the *opposite side.* The small letters a, b, c, written on sides opposite angles lettered with corresponding capital letters, represent the measures of the lengths of those sides.

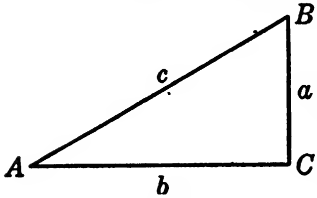


Fig. 49.

It has already been stated that the size of an angle does not depend upon the lengths of the sides that form the angle. However, a very close connection exists between the size of angle A and the value of any ratio that may be formed between the lengths of two sides of the triangle. The study of these ratios and the relations they bear to

the size of the angles is a part of the subject of trigonometry. In all, six different ratios can be formed between the measures of the lengths of the sides. They are

$$\frac{a}{c}, \quad \frac{b}{c}, \quad \frac{a}{b}, \quad \frac{c}{a}, \quad \frac{c}{b}, \quad \frac{b}{a}.$$

These ratios, of course, are abstract numbers. Names have been given to them. The first one, a/c, is called the *sine of angle* A, which is written in the shorter form

$$\sin A = \frac{a}{c}.$$

The sine of either acute angle of a right triangle is defined as the ratio of the opposite side to the hypotenuse. The sine of angle B is of course the ratio of b to c, written

$$\sin B = \frac{b}{c}.$$

To get an idea of the relation between the size of an angle and the value of one of these ratios, let us construct a right triangle whose sides and angles are known. For convenience, draw the line segment AD two units long and then construct on AD the equilateral triangle ADB. (See Fig. 50.) Each angle of triangle ADB contains 60°. Why? Now from B drop a perpendicular BC to the base AD. This perpendicular bisects the angle at B and bisects the base, resulting in the right triangle ACB, whose angles are all known and whose sides are all known except CB. CB is at once determined by use of the Theorem of Pythagoras:

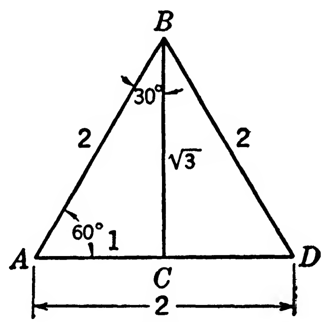


Fig. 50.

$$\overline{CB}^2 + \overline{AC}^2 = \overline{AB}^2.$$

$$\overline{CB}^2 + (1)^2 = (2)^2.$$

$$\overline{CB}^2 = 4 - 1.$$

$$\overline{CB}^2 = 3.$$

$$\overline{CB} = \sqrt{3}.$$

Now from the definition for the sine of an angle,

$$\sin A = \frac{CB}{AB} = \frac{\sqrt{3}}{2}.$$

Since A is 60°, we write

$$\sin 60° = \frac{\sqrt{3}}{2}.$$

If A were an acute angle different from 60°, this ratio could not have $\sqrt{3}/2$ as its value. Furthermore, if this particular ratio had a value different from $\sqrt{3}/2$, A could not be 60°. The point to be emphasized is that the sine of an angle of a certain size has a certain definite value. Note that if an angle is 30° its sine is $\frac{1}{2}$; $B = 30°$, and $\sin B = AC/AB = \frac{1}{2}$.

The values of the other ratios are just as dependent upon the size of angle A and independent of the size of the triangle as is the sine. For this reason, they are called functions of angle A. The ratio b/c is the cosine of angle A, written "cos A." *The cosine of an acute angle of a right triangle is defined as the ratio of the adjacent side to the hypotenuse. The tangent of* A, *written "tan* A,*" is the name given to the ratio* a/b, *which is the ratio of the opposite side to the adjacent side.* Notice that the other three ratios, c/a, c/b, and b/a, are reciprocals of the three just discussed, in the order mentioned. They are called the *cosecant of* A (csc A), the *secant of* A (sec A), and the *cotangent of* A (cot A), respectively. We shall have occasion to use only the sine, cosine, and tangent in this course. The student should memorize these definitions:

$$\text{Sine of an angle} = \frac{\text{Side opposite the angle}}{\text{Hypotenuse}}.$$

$$\text{Cosine of an angle} = \frac{\text{Side adjacent to the angle}}{\text{Hypotenuse}}.$$

$$\text{Tangent of an angle} = \frac{\text{Side opposite the angle}}{\text{Side adjacent to the angle}}.$$

By reference to the 30°, 60°, right triangle of Fig. 50 we may now write the values of the trigonometric functions of 60° and also of 30°.

Functions of 60°	*Functions of 30°*
$\sin 60° = \frac{\sqrt{3}}{2}$	$\sin 30° = \frac{1}{2}$
$\cos 60° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$
$\tan 60° = \sqrt{3}$	$\tan 30° = \frac{1}{\sqrt{3}}$
$\csc 60° = \frac{2}{\sqrt{3}}$	$\csc 30° = 2$
$\sec 60° = 2$	$\sec 30° = \frac{2}{\sqrt{3}}$
$\cot 60° = \frac{1}{\sqrt{3}}$	$\cot 30° = \sqrt{3}$

Would the values of these ratios have been different if the equilateral triangle had been constructed with a side whose length was other than two units?

Every right triangle which contains a 30° angle is *similar* to triangle ACB of Fig. 50 and must therefore have each of the ratios between two of its sides equal to the ratio between corresponding sides of triangle ACB. In fact, the fundamental basis for the development of trigonometry is this principle: *All right triangles containing a common-sized acute angle are similar and therefore have their corresponding ratios between lengths of sides equal.* This fact may suggest to the student the possibility of tabulating the values of these ratios for acute angles of various sizes.

Another angle whose functions can be determined by con-

struction is an angle of 45°. At any point C on a straight horizontal line erect a perpendicular. (See Fig. 51.) Now with C as a center and a radius of 1 (any other radius might be used) swing an arc cutting the horizontal line at A and the vertical line at B. Draw AB, thereby forming a right triangle with $A = 45°$, $a = 1$, $b = 1$, and $c = \sqrt{2}$. Why is $c = \sqrt{2}$?

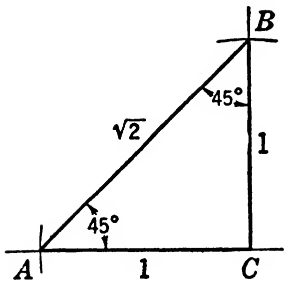


Fig. 51.

From the definitions of the functions, we have

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \text{ or } .707, \text{ approximately,}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \text{ or } .707, \text{ approximately,}$$

and

$$\tan 45° = \frac{1}{1} = 1.$$

The student should learn to construct the two triangles just discussed and from them give the values of the functions of 30°, 45°, and 60°. In expressing these values in decimal form he will find it convenient to remember that

$$\sqrt{2} = 1.414, \text{ approximately,}$$

and

$$\sqrt{3} = 1.732, \text{ approximately.}$$

It is not possible to find the values of the trigonometric functions of all angles by geometric constructions. However, by means of a protractor and a straight edge a fairly good drawing can be made of a right triangle which contains an acute angle of a particular size. A measuring scale can be used to determine approximately the lengths of the sides of the triangle; and the value of any function can be found approximately by forming the appropriate ratio. Such

drawings should be made large enough so that measurement of the sides is practicable.

Problems

1. Draw a right triangle ACB with side a 3 in. long and side b 4 in. long. How long is side c? Give the values of sin A, cos A, and tan A. By use of a protractor find approximately the size of A. If A is known, how can B be found? Find B.

2. Draw a right triangle ACB with side $a = 12$ and the hypotenuse $c = 13$. Find b. With a protractor find A. Find B. Give the values of sin B, cos B, and tan B.

3. In Fig. 52 find by actual measurements with ruler and protractor the approximate length of each side and the size of each angle. Then give the values of the three main functions of A.

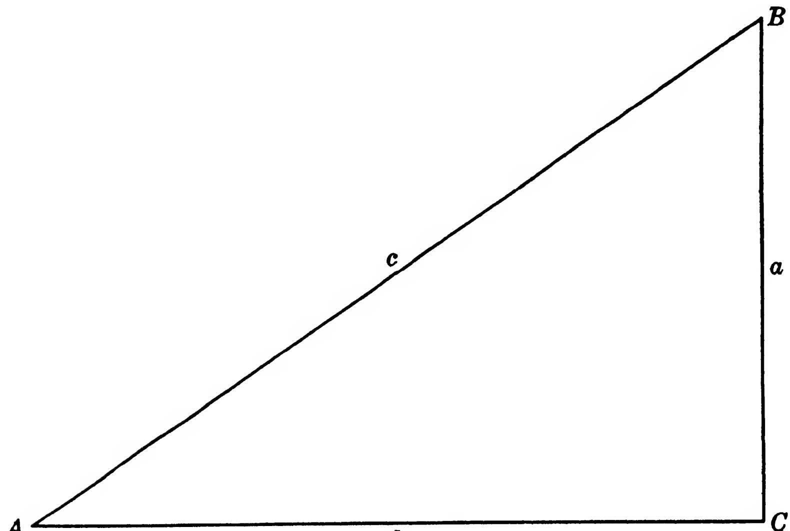


Fig. 52.

4. Using a protractor and straight edge to make a suitable drawing, find approximately the values of the sine, cosine, and tangent of 52°.

5. By the method used in Problem 4, find the values of the sine, cosine, and tangent of 20°.

6. Derive the values of the sine, cosine, and tangent of 45° from a right triangle each of whose legs is 3 in. long.

7. By drawing first an equilateral triangle whose side is 5 units long, form a 60°, 30°, right triangle and derive the values of the sine, cosine, and tangent of 60°. Also give the values of these functions of 30°.

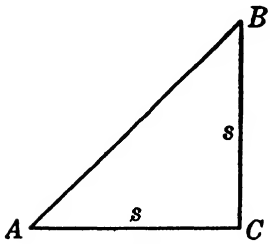


Fig. 53.

8. In Fig. 53 s is a constant representing the length of sides AC and CB. What is the size of $\angle A$? Why? Find AB in terms of s. Now find the values of the functions of 45°. Does it matter what value is given to s?

9. In Fig. 54 find a and b. *Suggestion:* $a/c = \sin 30°$. Sin $30° = \frac{1}{2}$, and $c = 28$. Therefore $a/28 = \frac{1}{2}$, from which a is easily determined. The side b is found by using cos 30°.

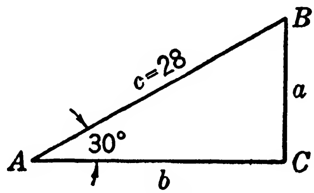


Fig. 54.

10. A guy wire from the top of a pole to a point on the ground 50 ft. from the foot of the pole makes an angle of 45° with the horizontal. How high is the pole? What is the length of the guy wire?

Determining the Values of the Trigonometric Ratios by Line Measurement

Fig. 55 may be used in determining approximately the values of the functions of an angle. The angle is formed with its vertex at A and adjacent side along AX. It may then be considered as a part of a right triangle whose hypotenuse is the radius of the circular arc. Since the hypotenuse AB is 100 units in length, the number of hundredths in the sine of the angle is numerically the same as the number of units in the length of the opposite side CB; and the cosine, expressed in hundredths, is numerically the same as the number of units in the length of the adjacent side AC. The tangent may be determined by dividing the opposite side by the adjacent side. The tangent can also be obtained by measurement of a single line if the right angle of the triangle is formed at C', as shown in the figure, the number

of hundredths in the tangent being the same numerically as the number of units in the length of $C'B'$.

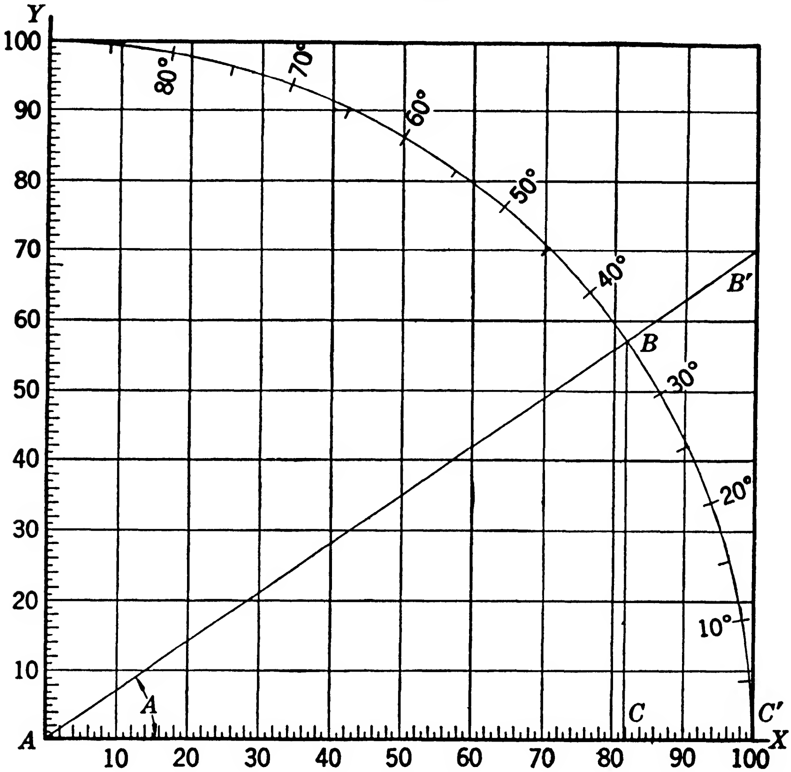


Fig. 55.

For example, in the figure, A is represented as being 35°. We have then

$$\sin 35^\circ = \frac{\text{Number of units in } CB}{100} = \frac{57}{100} = .57, \text{ approximately,}$$

$$\cos 35^\circ = \frac{\text{Number of units in } AC}{100} = \frac{82}{100} = .82, \text{ approximately,}$$

and

$$\tan 35^\circ = \frac{\text{Number of units in } CB}{\text{Number of units of } AC} = \frac{57}{82} = .70, \text{ approximately,}$$

or

$$\tan 35^\circ = \frac{\text{Number of units in } C'B'}{100} = \frac{70}{100} = .70, \text{ approximately.}$$

(Incidentally, note that $\tan A = \sin A/\cos A$ for any angle A.)

This diagram may also be used in estimating the size of an angle whose sine, cosine, or tangent is known.

Problems

Solve the following problems by reference to Fig. 55:

1. Determine approximately the values of the following functions: sin 20°, cos 20°, tan 20°, cos 60°, tan 50°, sin 80°, tan 10°, cos 5°, tan 65°, sin 15°.

2. What is the greatest value the sine of an angle can have? What is the angle whose sine has this value?

3. What is the greatest value that the cosine of an angle can have? What is the angle whose cosine has this value?

4. As an angle increases from 0° to 90° its sine increases from what value to what value?

5. As an angle increases from 0° to 90° its cosine has what range of values?

6. What range of values does the tangent of an angle have as the angle increases from 0° to 90°? Note that there is no tangent of 90°. Why?

7. Estimate the size of each of the angles represented here: $\sin A = .19$, $\tan R = .49$, $\cos M = .89$, $\tan B = 1.15$, $\sin T = .81$.

8. If the sum of two angles is 90°, each of the angles is said to be the complement of the other angle. What is the complement of 30°? Compare sin 30° with cos 60°.

9. Compare cos 20° with sin 70°.

10. In a right triangle ACB, note that $A + B = 90°$, and therefore A and B are complementary angles. How does $\sin A$ compare with $\cos B$? How does $\cos A$ compare with $\sin B$?

Use of Tables

The values of most of the trigonometric functions of angles cannot be written exactly as decimal numbers. However, it is possible by methods employed in more advanced mathematics to find the value of a function to a desired number of decimal places. For convenience, tables have been made

giving the values of functions to two, three, four, five, and even more decimal places. The table to be used in any case depends upon the accuracy required.

Table X gives to the nearest ten-thousandth the values of the trigonometric ratios of angles from 0° to 90°, angles being expressed at intervals of 10 min. The entries in the table are so arranged that angles from 0° to 45° are read *down* the first column and the names of the functions are given across the *top* of the table, while angles from 45° to 90° are read *up* the last column and the names of the functions are read across the *bottom* of the table. It thus appears that each entry in the table serves as the value of two different functions. Why is such an arrangement possible? The two functions are functions of angles that are related in what way?

If an angle is expressed in greater detail than is given in the table, the value of a function of the angle can be estimated by locating the angle between two successive angles listed in the table and reasoning that the value of the required function should lie between the values of the functions of these two angles.

Examples

1. Find sin 63° 45′.

An angle of 63° 45′ is not listed in the table. It seems reasonable that sin 63° 45′ should lie about halfway between sin 63° 40′ and sin 63° 50′.

$$\begin{array}{lrl} & \sin 63^\circ\ 50' = & .8975 \\ & \underline{\sin 63^\circ\ 40'} = & \underline{.8962} \\ \text{Differences:} & 10' & .0013 \end{array}$$

An *increase* of 10′ in the angle is accompanied by an *increase* of .0013 in the value of the sine. Then an *increase* of 5′ in the angle should be accompanied by an *increase* in the sine of about $\frac{5}{10}$ of .0013, or .5 × .0013 = .00065, which, to four decimal places, is taken as .0007. Therefore, we conclude that sin 63° 45′ = .8962 + .0007 = .8969, approximately.

2. Find cos 27° 38′.

Cos 27° 38′ lies between cos 27° 30′ and cos 27° 40′.

	$\cos 27° 40' =$	$.8857$
	$\cos 27° 30' =$	$.8870$
Differences:	$10'$	$.0013$

An *increase* of $10'$ in the angle is accompanied by a *decrease* of .0013 in the value of the cosine. Then an *increase* of $8'$ in the angle (from $27° 30'$ to $27° 38'$) should be accompanied by a *decrease* in the cosine of about $\frac{8}{10}$ of .0013, or .00104, which, to four decimal places, is taken as .0010. Hence $\cos 27° 38' = .8870 - .0010 = .8860$, approximately.

3. If $\tan A = 3.0062$, what is A?

The number 3.0062 is not listed in the table as a value of the tangent. However, it lies between the two numbers 2.9887 and 3.0178, which are listed as the values of $\tan 71° 30'$ and $\tan 71° 40'$, respectively.

	$3.0178 =$	$\tan 71° 40'$	3.0062
	$2.9887 =$	$\tan 71° 30'$	2.9887
Differences:	0.0291	$10'$	0.0175

An *increase* of .0291 in the tangent corresponds to an *increase* of $10'$ in the angle. Then an *increase* of .0175 in the tangent should correspond to an increase of approximately $\frac{.0175}{.0291}$ of $10'$, or $\frac{175}{291} \times 10'$, which is about $6'$. Hence $A = 71° 30' + 6' = 71° 36'$, approximately.

Problems

Solve the following problems by reference to Table X:

1. Give the value of $\tan 18°$, $\cos 5°$, $\sin 41°$, $\cos 32° 20'$, $\sin 22° 50'$, $\tan 43° 10'$.

2. Give the value of $\sin 45°$, $\tan 45°$, $\cot 45°$, $\sin 60°$, $\tan 60°$, $\cos 60°$, $\cos 60° 20'$.

3. Give the value of $\sin 68°$, $\cos 82°$, $\tan 76°$, $\cos 73° 30'$, $\sin 56° 10'$, $\tan 45° 40'$.

4. Give the value of $\sin 0°$, $\sin 90°$, $\cos 0°$, $\cos 90°$, $\tan 0°$, $\tan 89°$.

5. Find $\cos 35° 30'$, $\sin 43° 18'$, $\tan 12° 56'$, $\cos 39° 7'$.

6. Find $\tan 55° 15'$, $\cos 57° 43'$, $\tan 80° 19'$, $\sin 89° 14'$.

7. Find $\sin 34° 40'$, $\tan 0° 54'$, $\cos 0° 45'$, $\sin 67° 28'$.

8. Find $\cos 44° 25'$, $\sin 75° 3'$, $\cos 51° 37'$, $\tan 89° 12'$.

9. What angle has its sine equal to .6472?

10. If $\sin A = .8572$, what is A? If $\tan B = .1673$, what is B?

11. If $\cos A = .8718$, what is A? If $\tan R = 4.5736$, what is R?

12. Determine the angle involved in each of the following: $\sin M = .3297$; $\tan A = .4428$; $\cos B = .4908$; $\sin A = .9360$; $\cos D = .9055$.

13. A 20-ft. ladder leaning against a vertical wall makes an angle of 70° 15′ with the ground. How high up the wall does the ladder reach?

14. How long a rafter is required for a roof whose span is 30 ft. if the roof makes an angle of 33° with the horizontal?

15. Note that the pitch of a roof, as previously defined, may be considered as one-half of the tangent of the angle that the roof makes with the horizontal. For a roof whose pitch is $\frac{1}{2}$, what size is this anglé?

16. About what angle does a roof whose pitch is $\frac{1}{4}$ make with the horizontal?

17. How high is a kite that is at the end of a 500-ft. taut string making an angle of 23° 40′ with the ground?

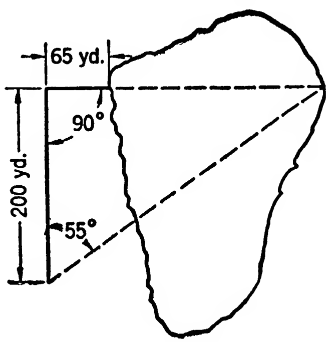


Fig. 56.

18. From a point on the ground 40 ft. from the foot of a certain tree an observer finds the angle of elevation of the top of the tree to be 50° 38′. How tall is the tree? (The angle of elevation is the angle which the line of sight makes with the horizontal.)

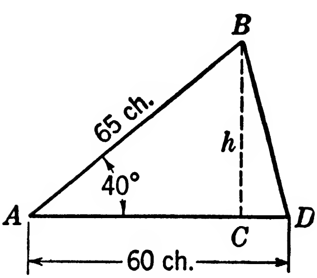


Fig. 57.

19. The side of a hill is inclined 6° 30′ from the horizontal. How many feet does one rise in walking 200 ft. up the side of the hill?

20. In order to determine the greatest width of a lake the measurements indicated in Fig. 56 were made. Find the width of the lake.

21. The triangle shown in Fig. 57 is not a right triangle, but, if

the altitude is dropped from B to the base, two right triangles are formed. The length, h, of altitude CB can then be determined. Find h. Find the area of the triangle.

Area of a Triangle

By the method employed in Problem 21 of the preceding group a formula can be developed for finding the area of any triangle in which two sides and the included angle are known. Consider the triangle ADB in Fig. 58, in which we may suppose that A, b, and d are known. Draw the altitude CB. Now,

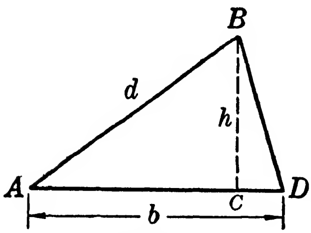


Fig. 58.

$$\text{area} = \tfrac{1}{2}bh$$

and

$$\frac{h}{d} = \sin A,$$

from which

$$h = d \sin A.$$

Therefore

$$\text{area} = \tfrac{1}{2}bd \sin A.$$

In the case just considered all angles of the triangle are acute. Suppose that A is obtuse, as shown in Fig. 59. Extend the base through A and draw altitude CB. Now,

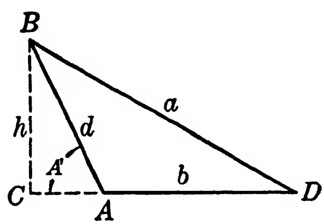


Fig. 59.

$$\text{area} = \tfrac{1}{2}bh$$

and

$$\frac{h}{d} = \sin A',$$

or

$$\frac{h}{d} = \sin (180° - A),$$

from which

$$h = d \sin (180° - A).$$

Therefore, if A is obtuse,

$$\text{area} = \tfrac{1}{2}bd \sin (180° - A).$$

The angle $(180° - A)$ is called the *supplement* of angle A. In a more extensive treatment of trigonometry it is shown that the sine of an obtuse angle is the same as the sine of its supplement. That is, $\sin A = \sin (180° - A)$.

Hence the formula for the area of a triangle may be stated in words as follows:

The area of any triangle is equal to one-half the product of any two sides times the sine of their included angle.

Problems

1. Find the number of acres in the triangular plot of land represented by Fig. 60.

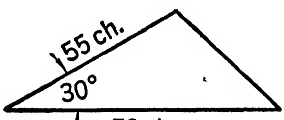


Fig. 60.

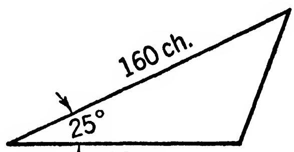


Fig. 61.

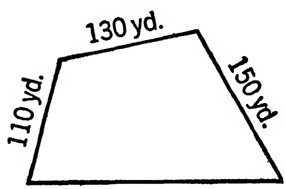


Fig. 62.

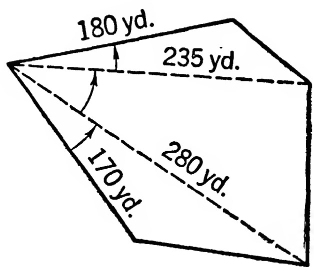


Fig. 63.

2. Find the area of the piece of land represented by Fig. 61.

3. Find the number of acres in the four-sided plot of land represented by Fig. 62. (*Suggestion:* Divide the quadrilateral into two triangles and measure the angles needed for use of the formula just developed.)

4. To get the area of the five-sided field represented by Fig. 63 all of the angles required were read on an instrument at one corner of the field and then the indicated measurements of distance were made. How many acres are there in the field?

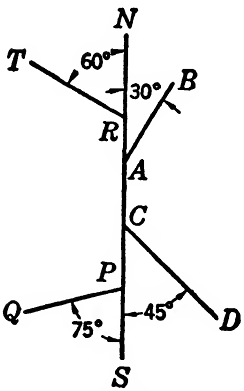


Fig. 64.

Bearing of a Line

In descriptions of tracts of land from measurements of angles and distances, the bearing (that is, the direction) of a line from a given point is expressed in terms of the acute angle that the line makes with a meridian (a north and south line) drawn through the given point. Thus in Fig. 64 the line *NS* is a meridian; the bearing of *AB* is N. 30° E., which is read "north 30° east"; the bearing of *CD* is S. 45° E.; that of *PQ* is S. 75° W.; and that of *RT* is N. 60° W.

Problems

1. In the following description the bearings and lengths of the boundary lines of a certain farm are given: "Beginning at a point known as the northwest corner of the tract, same being marked by a 2-in. iron pipe, and running S. 0° 50′ E. 1975 ft., thence S. 20° 30′ E. 970 ft., thence N. 77° 15′ E. 1130 ft., thence N. 19° 50′ W. 475 ft., thence N. 68° E. 1255 ft., thence N. 53° 20′ W. 1635 ft., thence N. 26° 45′ E. 1810 ft., thence S. 65° 45′ W. 2150 ft. to beginning corner." Using a protractor, straight edge, and measuring scale, draw a map of this farm using a scale of 1 in. to 660 ft.

2. Determine approximately the number of acres contained in the farm described in Problem 1. (The result should be about 119 A.)

3. The boundary lines of a certain tract of land are as indicated in Fig. 65. Determine the scale used and write a description of the tract, beginning at the southwest corner, proceeding around the tract in a clockwise direction, and giving the bearing and length of each of the boundary lines.

4. On the farm pictured in Fig. 66 find approximately the number of acres of land in each of the fields lettered. The scale is 1 in. to 660 ft.

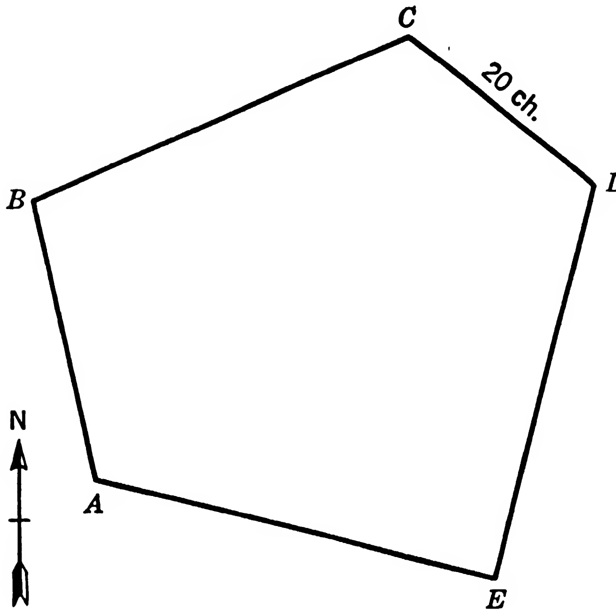
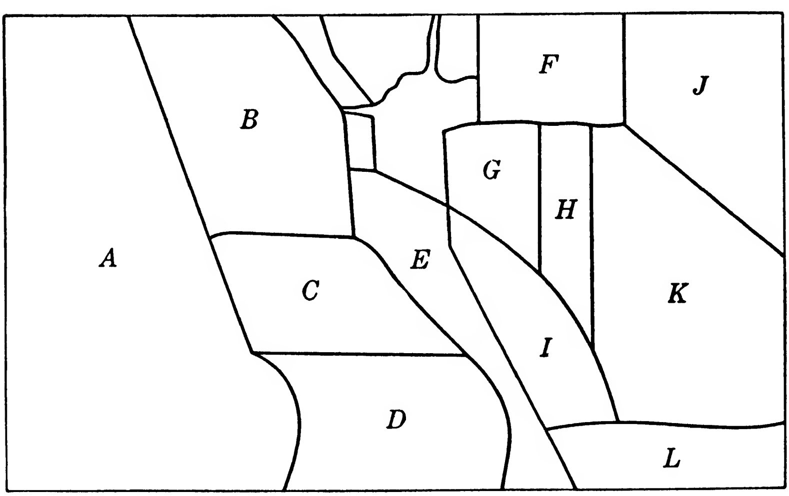


Fig. 65.

5. Assuming that the diagonal from the lower right corner of Fig. 66 to the upper left corner extends due north, write a description of the tract of land pictured, indicating lengths and bearings of all boundary lines.



CHAPTER 7

Averages

A study of some particular feature or attribute of an item or of a group of items may be placed on a mathematical basis if a suitable collection of numbers that serve as measures of the property under consideration is available. In connection with such a study it is usually desirable to obtain one or more numbers which, with regard to the particular feature under study, are representative of the entire collection of measures. Representative numbers of this sort are called *averages*.

The kinds of averages most frequently employed are (1) the arithmetic average, (2) the mode, (3) the median, (4) the geometric mean, and (5) the harmonic mean. The kind of average to be used in a particular case depends upon what the average is intended to show. A brief discussion of each of these averages may suggest to the student its appropriate use.

The Arithmetic Average

The arithmetic average of a collection of measures is the quotient obtained by dividing the sum of the measures by the number of measures. If some of the measures occur more than once in the collection, a distinction should be made between the *simple arithmetic mean,* in which each of the different measures is counted only once in obtaining the sum of measures, and the *weighted arithmetic mean,* in which each of the different measures is multiplied by the number of times it occurs and the sum of the products so obtained is regarded as the sum of the measures. A particular weighted arithmetic mean is equivalent to the simple arith-

metic mean obtained by counting every measure as many times as it occurs in the given set of data. The averages computed in the following example illustrate appropriate uses of these two forms of the arithmetic average.

Example

A certain farmer has the following record of performance for a year on his herd of five milk cows:

Cow Number	Number of Pounds of Milk	Per Cent of Butterfat
1	3512	4.9
2	2478	4.1
3	9640	3.6
4	3250	5.4
5	11,265	3.5

What is the average production of milk per cow for the year? The answer to this question is the simple arithmetic mean of the measures given in the second column. It is obtained by dividing the total number of pounds of milk by the number of cows:

$$30{,}145 \div 5 = 6029.$$

We may say then that this herd averaged 6029 lb. of milk per cow for the year.

Next, consider the problem of finding the average per cent of butterfat in the milk produced by this group of cows. If the sum of the per cents for the individual cows is divided by 5, the result is 4.3%. Is 4.3% representative of the butterfat content of all the milk produced? Does not this figure give too much weight to the high quality of milk produced by cows 1 and 4, whose production was comparatively low, and too little consideration to the quantity of milk, testing low in butterfat, produced by cows 3 and 5? Note that the milk from these two cows made up over two-thirds of the total quantity produced and that the per cent of butterfat was 3.6 for one of them and 3.5 for the other.

Obviously, a more representative figure for the average per cent of butterfat in all of the milk would be obtained by dividing the total number of pounds of butterfat by the total number of pounds of milk. The total number of pounds of butterfat produced is the sum of the products obtained by multiplying the

production of milk for each cow by the corresponding per cent of butterfat. These products are shown below.

Cow Number	Computation	Number of Pounds of Butterfat
1	.049 × 3512	172.088
2	.041 × 2478	101.598
3	.036 × 9640	347.040
4	.054 × 3250	175.500
5	.035 × 11,265	394.275
Total	30,145	1190.501

If 1190.501 is divided by 30,145, the result is approximately .039, or 3.9%. This average of 3.9% is a weighted arithmetic mean. It is weighted in the sense that each of the per cents given for the individual cows is multiplied by the number of pounds of milk testing that per cent. For instance, the 4.9% (for cow No. 1) is said to have a weight of 3512, since .049 lb. of butterfat is contained in each of 3512 lb. of milk.

Computation of the arithmetic mean M of a set of n measures $m_1, m_2, m_3, \ldots, m_n$ may in some cases be shortened by selecting a number M' near which the true mean appears to be and using the relation developed in the following.

By definition,

$$M = \frac{m_1 + m_2 + m_3 + \ldots + m_n}{n}.$$

Subtracting M' from both sides, we obtain

$$M - M' = \frac{m_1 + m_2 + m_3 + \ldots + m_n}{n} - M'$$

$$= \frac{(m_1 + m_2 + m_3 + \ldots + m_n) - nM'}{n}.$$

In the preceding line, there are n terms inside the parentheses and there are the same number of M''s to be subtracted. Hence the equation can be written

$$M - M' = \frac{(m_1 - M') + (m_2 - M') + (m_3 - M') + \ldots + (m_n - M')}{n}.$$

The differences $(m_1 - M')$, $(m_2 - M')$, and so on, are readily obtained and are usually small in comparison to the measures whose average is sought. Note that the average of these differences is the difference between the true mean M and the approximate mean M'.

Example

Find the average weight per bale of 8 bales of lint cotton whose weights in pounds are 510, 495, 488, 506, 496, 525, 503, and 515.

For convenience, let $M' = 500$. Then the average of the differences, 10, -5, -12, 6, -4, 25, 3, and 15, is $\frac{38}{8}$, or $4\frac{3}{4}$. Therefore the average weight per bale for this lot of cotton is $500 + 4\frac{3}{4}$, or $504\frac{3}{4}$ lb.

The Mode

The mode of a group of measures is defined as that measure which occurs most frequently. In the above example, the mode of the number of pounds of butterfat present in each pound of all the milk produced is .035, since .035 lb. of butterfat is present in each of 11,265 lb. of milk. In other words, we may think of the measure .035 as occurring 11,265 times, which is the greatest number of times any one measure occurs in the set of observations. In the popular sense, mode means fashion; and as a mathematical average it has a similar meaning, since it indicates the most usual occurrence. For example, to say that the average age at which students enter college is 18 yr. means that there are more students entering college at that age than at any other age.

A given group of measures may exhibit more than one modal value. For example, in considering all the wages paid a large number of workmen engaged on a certain construction project, it might be observed that one modal wage is the most common wage paid to skilled workmen, while another modal wage is the most common wage paid to unskilled laborers.

The Median

The median is the middle measure of a group of measures. It is obtained by arranging the measures according to magnitude or size, counting each different measure as many times as it occurs, and then selecting the middle number of the group. In case the group contains an even number of measures, the median is the arithmetic mean of the two measures nearest the middle. In a set of observations there are just as many measures that are less than the median as there are that are greater. The example discussed above is not well adapted to showing an appropriate use of the median. The median production of milk per cow is 3512 lb.; the median of the per cents of butterfat is 4.1%; but with so few observations at hand, these figures are not significant in portraying any features typical of the two groups of measures. The median is useful in situations requiring arrangement according to size.

The Geometric Mean

The geometric mean of n numbers is the nth root of the product of the numbers. For example, the geometric mean of 3, 6, 12, and 24 is

$$\sqrt[4]{3 \times 6 \times 12 \times 24}.$$

The geometric mean of two numbers, often called the *mean proportional* between the two numbers, is simply the positive square root of their product. In the proportion

$$\frac{5}{x} = \frac{x}{45}$$

x is the geometric mean of 5 and 45. Solving this equation, we have

$$x^2 = 225,$$

from which

$$x = \sqrt{225} = 15.$$

That is, 15 is the geometric mean between 5 and 45.

The geometric mean of a set of measures is related to the product of the measures in a way analogous to the relation of the arithmetic mean to the sum of the measures. The sum of n terms each of which is the arithmetic mean of a set of n measures equals the sum of the measures, while the product of n factors each of which is the geometric mean of n measures equals the product of the measures. Also, the sum of the deviations from the arithmetic mean of measures that are exceeded by the mean is numerically the same as the sum of the deviations from the mean of the measures that exceed the mean, while the product of the ratios of the geometric mean to the measures that it exceeds is the same as the product of the ratios to the geometric mean of the measures that exceed it.

The property just mentioned makes the geometric mean useful as an average of ratios. For example, if shelling tests on two samples of corn show that for one sample the weight of grain is $\frac{8}{9}$ of the weight of ear corn and for the other the grain weight is $\frac{49}{72}$ of that of the ear corn, the average ratio of grain weight to ear corn weight for the two kinds of corn represented by the samples may be taken as $\sqrt{\frac{8}{9} \times \frac{49}{72}}$, which is $\frac{7}{9}$. Since per cents are ratios, the geometric mean is appropriately used as an average of per cents, particularly if the facts upon which the per cents are based are not available. For example, if two investments give annual yields of 3.2% and 5%, an average yield on the two investments is $\sqrt{3.2 \times 5}\%$, or 4%.

Computation of the geometric mean of more than two measures is usually effected by means of logarithms. Since logarithms have not yet been discussed in this text, the student may at this point indicate geometric means required in problems without carrying out the actual computations.

The Harmonic Mean

The harmonic mean of a set of measures is the reciprocal of the arithmetic mean of the reciprocals of the measures.

If H denotes the harmonic mean of the n measures m_1, m_2, $m_3, \ldots, m_n$, then H is given by the formula

$$\frac{1}{H} = \frac{\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} + \cdots + \frac{1}{m_n}}{n}.$$

The harmonic mean is appropriately used in averaging certain types of rates. For example, if an automobile travels a distance of 100 mi. at the rate of 50 mi. per hour and makes the return trip at the rate of 30 mi. per hour, the average rate of speed for the entire trip is the harmonic mean of 50 and 30 and not their arithmetic mean. In this case,

$$\frac{1}{H} = \frac{\frac{1}{30} + \frac{1}{50}}{2}$$

$$= \frac{\frac{8}{150}}{2}$$

$$= \frac{4}{150}.$$

Hence

$$H = \frac{1}{\frac{4}{150}} = \frac{150}{4} = 37.5.$$

The average speed for the entire trip is 37.5 mi. per hour. Note that this result is consistent with the fact that a total distance of 200 mi. was traveled in $5\frac{1}{3}$ hr.

As another example of the harmonic mean, consider the average number of pounds of grain that \$1 will buy if, of the only three kinds under study, one kind sells at 40 lb. for \$1, another sells at 25 lb. for \$1, and a third sells at 50 lb. for \$1. The arithmetic average of 40, 25, and 50 is $38\frac{1}{3}$. However, the three kinds of grain may be considered as priced at $2\frac{1}{2}$¢, 4¢, and 2¢ per pound. The average of these prices is $2\frac{5}{6}$¢, and at $2\frac{5}{6}$¢ per pound \$1 will buy $35\frac{5}{17}$ lb. This result, which seems to represent better than does $38\frac{1}{3}$ the average number of pounds purchasable at \$1, is really the harmonic mean of 40, 25, and 50.

$$\frac{1}{H} = \frac{\frac{1}{40} + \frac{1}{25} + \frac{1}{50}}{3}$$

$$= \frac{\frac{5 + 8 + 4}{200}}{3}$$

$$= \tfrac{17}{600}.$$

Hence

$$H = \tfrac{600}{17} = 35\tfrac{5}{17}.$$

The Standard Deviation

The significance of an average of a set of measures depends upon how representative it is of the whole set—that is, whether the numbers of the set vary greatly from it or fall within a comparatively narrow range about it. Some kind of average of the variations of the measures from a mean is usually sought. The most useful average of this type is the *standard deviation*, which is defined as the square root of the arithmetic mean of the squared deviations of the measures from their arithmetic mean. If M is the arithmetic average of n measures $m_1, m_2, m_3, \ldots, m_n$, then the standard deviation, denoted by the Greek letter sigma (σ), is given by the formula

$$\sigma = \sqrt{\frac{(m_1 - M)^2 + (m_2 - M)^2 + (m_3 - M)^2 + \ldots + (m_n - M)^2}{n}}.$$

Example

Find the standard deviation of the numbers 5, 7, 9, 12, 15, and 18.

The arithmetic mean in this case is 11, and the deviations from it are (-6), (-4), (-2), 1, 4, 7. The average of the squares of these numbers is

$$\frac{36 + 16 + 4 + 1 + 16 + 49}{6}, \text{ or } 20.3333, \text{ approximately.}$$

Hence

$$\sigma = \sqrt{20.3333} = 4.51, \text{ approximately.}$$

It should be admitted that the foregoing discussion of averages falls far short of covering the subject. Measures involved in sets of data have been treated as if they were exact, whereas, in actual practice, measures are usually approximations within certain ranges or class intervals. If a given set of measures is grouped into equally spaced class intervals and the calculation of averages is based upon the distribution of the measures over the various class intervals, methods more complex than those set forth above are required.

Problems

1. What is the average weight of 5 steers weighing 320 lb., 516 lb., 437 lb., 328 lb., and 469 lb.?

2. What is the average price per dozen received for eggs if 3 doz. are sold at 20¢ per dozen, 4 doz. at 18¢, 7 doz. at 15¢, and 2 doz. at 24¢? What kind of average is this?

3. The following table shows the production of cotton for a certain year in a community comprising eight farms:

Farm Number	Number of Acres in Cotton	Total Number of Pounds of Lint Produced
1	75	11,250
2	43	5590
3	6	2892
4	180	36,900
5	9	4077
6	256	28,672
7	63	8820
8	50	6750

The average yield per acre for each farm is what kind of average? Find it for each farm.

4. The average yield per acre for the entire community in Problem 3 is what kind of average? Find it.

5. What is the average number of acres in cotton per farm in the community in Problem 3?

6. Suppose that in compliance with a program for reducing cotton acreage the following per cent reductions are made in the

farms in Problem 3: farms 2, 3, and 5, 25%; farms 1, 7, and 8, 30%; and farms 4 and 6, 35%. What is the average per cent of reduction in cotton acreage in the community?

7. In a random sample from a crib of corn the lengths of 10 ears in inches are 8, 5, 7, 11, 7, 8, 9, 6, 10, and 7. What is the average length of an ear in this sample? What is the median length of ear? What is the most common, or modal length?

8. Twenty steers were grouped and bought as follows: 4 head at $10 per head, 6 head at $12, 8 head at $15, and 2 head at $20. What was the average price paid per head?

9. In 10 years a corn planter depreciated in value from $75 to $15. What was the average annual depreciation?

10. A lot of pigs were grouped and sold as follows: 8 head at $6 per head, 10 head at $7.50, 4 head at $9, and 3 head at $10. What was the average price per head?

11. Assuming that the letters *a*, *b*, and *c* have definite numerical values, write a formula (1) for their arithmetic average, (2) for their geometric mean, and (3) for their harmonic mean.

12. In a class of 23 students, the following grades were made in a certain subject: two A's, eight B's, six C's, four D's, two E's, and one F. What was the median letter grade obtained?

13. Assuming that the letter grades mentioned in Problem 12 have definite numerical values, write a formula for the average grade.

14. A farmer has the following record on the yield of lint from 5 bales of seed cotton:

Bale Number	Pounds of Seed Cotton	Pounds of Lint Cotton
1	1640	523
2	1625	512
3	1575	508
4	1600	500
5	1530	490

Determine the per cent yield of lint for each bale and the average per cent yield of lint for the five bales.

15. If the total amount received for the 5 bales of cotton mentioned in Problem 14 was $316.63, what was the average price

received per bale? What was the average price received per pound of lint?

16. What is the average price per pound received for cotton if one 490-lb. bale is sold at 12¢ per pound, two 510-lb. bales are sold at 14¢ per pound, and three 500-lb. bales are sold at 15¢ per pound. What is the average price received per bale?

17. What was the average price received per dozen for eggs if 9 doz. were sold at 20¢ per dozen, 12 doz. were sold at 18¢, 21 doz. were sold at 15¢, and 6 doz. were sold at 24¢?

18. What was the average price received per dozen for eggs if 8 doz. were sold at 40¢ per dozen, 5 doz. were sold at 35¢, 7 doz. were sold at 30¢, and 12 doz. were sold at 28¢?

19. What is the average price paid per pound of plant food nutrient if 3 T. of a 2-8-4 fertilizer are bought at $30 per ton?

20. What is the average price paid per hundredweight for cottonseed meal if 2 T. are bought at $45 per ton, 1 T. is bought for $40, and 3 T. are bought at $38 per ton?

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21. On a certain construction job 5 men each receive a daily wage of $2.50, 6 men each receive $4.25, 8 men each receive $9, and 1 man receives $12. What is the average daily wage per man on this job?

22. What is the average price per pound received for a 300-lb. beef carcass if 10% of it is sold at 25¢ per pound, 18% at 32¢, 24% at 30¢, 22% at 22¢, 14% at 18¢, 9% at 15¢, and 3% at 10¢.

23. What was the average price per pound received by a farmer for potatoes if he sold 18 bu. at $2.50 per bushel, 12 bu. at $3.20 per bushel, and 1500 lb. at $3\frac{1}{2}$¢ per pound?

24. What was the average price per quart received by a dairyman for milk if he sold 12 gal. at 36¢ per gallon, 75 qt. at 10¢ per quart, 40 pt. at 7¢ per pint, and 24 half-pints at 4¢ per half-pint?

25. A man accepted a job on January 1 of a certain year at $150 per month, payable at the end of the month. After 3 mo. his salary was increased by $25 per month; at the end of the next 3 mo. he received an additional increase of $15 per month; and at the end of each 2 mo. thereafter he received a further increase of $10 per month. How much did he receive for his work in December of that year? What was his average monthly salary for the year?

26. A group of 5 men worked 12 da. of 8 hr. each on a certain job at 60¢ an hour and 18 da. at 80¢ an hour. A group of 7 men working on the same job received $8 per day per man during the first 12 da. and $9 per day per man during the next 18 da. What was the average amount received per hour by a man on this job during the 30-da. period?

27. Find the geometric mean of 2 and 8.

28. Find the geometric mean of $6\frac{3}{4}$ and 12.

29. Find the geometric mean of 2, $5\frac{1}{3}$, and 6.

30. Find the geometric mean of 3, 6, 8, and 9.

31. During a certain year the prices of three basic commodities increased 3%, 8%, and 9% of their respective average prices for the preceding year. Determine the average per cent of increase in price for these commodities.

32. It is estimated that during the first year of its use a certain machine depreciates 16% of its cost value and during the second year it depreciates 9% of the value that it had at the beginning of that year. Find the average annual per cent of depreciation.

33. The numerical grades made by a class of 20 students in a certain course were 86, 82, 70, 65, 75, 70, 95, 68, 77, 77, 72, 65, 73, 77, 84, 67, 55, 76, 88, and 76. Find the median grade and use it in determining the range of grades to be designated by the letter C if this median grade is also the median of the C's and 50% of all the grades are to be C's.

34. Show that the harmonic mean H of two numbers a and b is given by the formula $H = 2ab/(a + b)$.

35. Show that the harmonic mean H of three numbers a, b, and c is given by the formula $H = 3abc/(ab + ac + bc)$.

36. On one occasion a farmer plows a field of 20 A. at the rate of $1\frac{1}{4}$ A. per hour, and during a second plowing he averages $2\frac{1}{2}$ A. per hour. What is his average rate of plowing in acres per hour?

37. A man travels a distance of 300 mi. by train at an average rate of 40 mi. per hour and returns an equal distance by automobile at an average rate of 50 mi. per hour. What was his average rate of travel for the whole trip?

38. On the basis of average prices of fertilizer materials for a certain year it is estimated that $1 will buy 8 lb. of nitrogen or

12 lb. of phosphoric acid or 16 lb. of potash. What is the average number of pounds of plant food that $1 will buy?

39. Find to two decimal places the standard deviation of the following numbers of pounds in the weights of ten 1-bu. baskets of wheat: 58, 60, 61, 59, 61, 62, 60, 59, 61, 59.

40. Laboratory tests on eight samples of cottonseed meal supposed to contain 43% protein showed their protein contents by per cents to be as follows: 42.6, 43.0, 41.8, 43.6, 44.0, 43.2, 43.8, 42.8. Find the standard deviation of these measures.

CHAPTER 8

Graphs

It is often desirable to use diagrams or scaled drawings to present quickly and forcefully the main facts indicated by a collection of data. Such drawings are commonly called *graphs*. The student has met with many graphical presentations of statistics in his study of geography and other subjects and also in present-day newspapers and magazines. The kinds of graphs most frequently used are bar graphs, area charts, and rectangular coördinate, or line, graphs.

Bar Graphs

The *bar graph* of a set of data is simply a set of line segments or bars whose lengths are proportional to the measures they represent. In constructing a bar graph, a correspondence is first set up between a unit of length of the bar and some definite measure used in the data to be presented. Bar graphs are particularly useful in exhibiting comparisons between measures of items of the same kind.

As an example of the use of the bar graph, consider the data in the accompanying table.

DAILY WATER REQUIREMENTS OF ANIMALS

Animal	Gallons of Water per Day
Horse	9
Cow	$8\frac{1}{2}$
Hog	$2\frac{1}{2}$
Sheep	$1\frac{1}{2}$

If we let 1 in. represent 2 gal. of water, these data may be exhibited graphically as shown in Fig. 67.

The cumulative type of bar graph, in which each bar is made up of sections that correspond to parts that make up the measure represented by the whole bar, is illustrated by Fig. 68. The data em-

ployed in making this graph are on page 97. The following interpretation of the graph is given in the 1941 Agri-

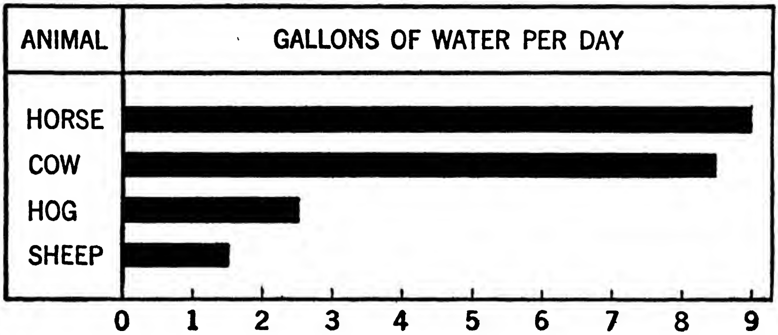


Fig. 67.

cultural Outlook Chart on Livestock, published by the Bureau of Agricultural Economics, United States Department of Agriculture:

During the 10 years prior to 1934 the annual pig crop of the United States averaged about 78 million head, of which nearly 75 per cent was produced in the Corn Belt States. Because of

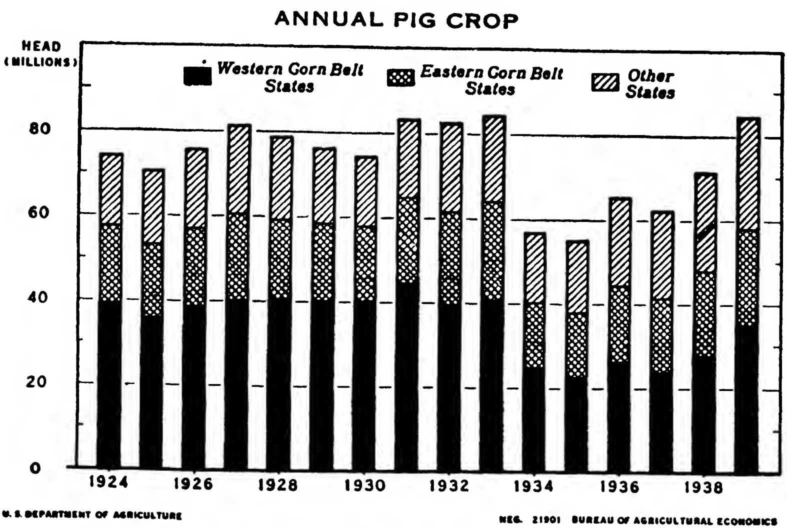


Fig. 68.

drought conditions in 1934 which greatly curtailed corn production, the pig crops of 1934 and 1935 were greatly reduced. Some increase occurred in 1936 but dry weather again in that year caused another reduction in the pig crop in 1937. With the return of normal weather conditions and increased feed production in the Corn Belt, pig crops have again increased. The 1939 pig crop of 84.3 million head was the largest crop on record. Since late 1939, however, hog prices have been low relative to the price of corn, and this has been reflected in reduced pig crops in all regions of the United States in 1940.

Annual Pig Crop, by Regions, United States, 1924–1939
(in thousands)

Year	Eastern Corn Belt	Western Corn Belt	Total Corn Belt	Other States	United States Total
1924	18,512	39,128	57,640	16,425	74,065
1925	17,433	35,955	53,388	16,922	70,310
1926	18,428	38,704	57,132	18,312	75,444
1927	20,015	40,236	60,251	20,995	81,246
1928	18,974	40,382	59,356	19,326	78,682
1929	18,247	40,229	58,476	17,649	76,125
1930	17,881	40,025	57,906	16,229	74,135
1931	19,886	44,651	64,537	18,639	83,176
1932	21,836	39,487	61,323	21,202	82,525
1933	23,022	40,670	63,692	20,508	84,200
1934	15,445	25,025	40,470	16,296	56,766
1935	15,442	22,646	38,088	16,998	55,086
1936	18,081	26,376	44,457	20,460	64,917
1937	17,860	23,581	41,441	20,466	61,907
1938	20,106	27,866	47,972	23,129	71,101
1939	23,478	34,312	57,790	26,538	84,328

Modifications of the bar graph are sometimes made by having the bars consist of pictures suggestive of the items represented. Such modifications tend to make the presentation more vivid and effective, as will be seen from Figs. 69 and 70, which were taken from a United States Department of Agriculture yearbook.

Note that in Fig. 69 the bags differ only in height, and so do the barrels. Pictorial diagrams in which the objects

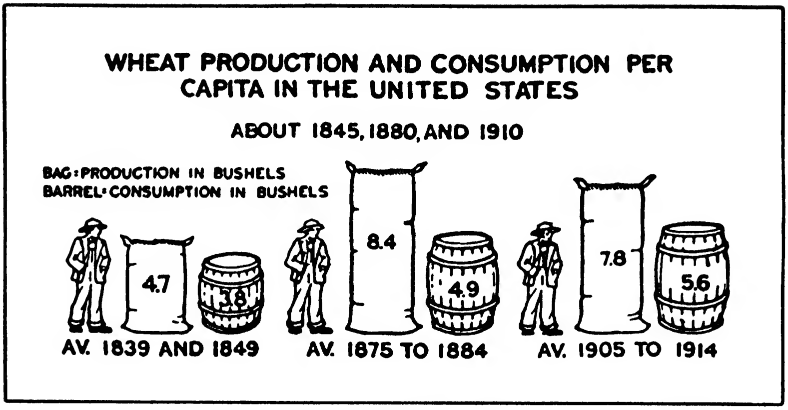


Fig. 69.

vary in more than one dimension are subject to misinterpretation. The type of pictorial representation illustrated in Fig. 70 is probably more readily interpreted.

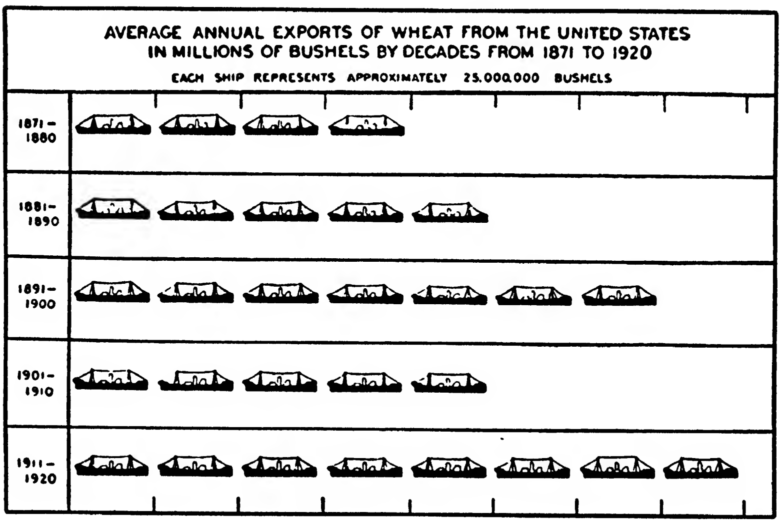


Fig. 70.

Problems

1. The number of pounds of nitrogen required per acre for producing certain crops is as follows: 10 for corn, 12 for wheat, 40 for cabbage, 45 for onions, 15 for radishes, 30 for potatoes, 5 for beans. Prepare a bar graph showing these data.

2. World's corn crop, by continents (5-yr. average, 1930-1934): North America, 2,403,000,000 bu.; South America, 563,064,000 bu.; Europe, 826,000,000 bu.; Africa, 170,812,000 bu.; Australia, 7,150,000 bu.; Asia, 464,000,000 bu. Prepare a bar graph showing these data.

3. The approximate average lengths of fibers of the principal kinds of cotton are as follows: Sea Island, 1.61 in.; Egyptian, 1.41 in.; American Upland, 0.93 in.; American long-staple, 1.3 in. Prepare a bar graph showing these data.

4. The estimated number of goats clipped in 1934 in the principal mohair-producing states was as follows: Texas, 2,795,000; New Mexico, 220,000; California, 35,000; Arizona, 150,000; Missouri, 72,000; Oregon, 87,000. Prepare a bar graph showing these data.

5. The estimated farm value per head, in 1935, of horses on farms of the South Central States was as follows: Tennessee, $81; Kentucky, $79; Alabama, $73; Mississippi, $64; Arkansas, $56; Louisiana, $45; Oklahoma, $57; Texas, $51. Prepare a bar graph showing these data.

6. From a study of costs of food per family for Pennsylvania and Ohio farm families of various sizes, each having a total annual income between $750 and $999, the estimates given in the following table were made. Prepare a bar graph with cumulative bars to exhibit these data, representing the value of food purchased by one portion of each bar and the value of home-produced food by another portion.

Number of Persons in Family	Value of Food Consumed Annually per Family		
	Purchased	Home-produced	All
2	$120	$220	$340
3	150	240	390
4	170	270	440
5	180	300	480
6	190	320	510
7	200	327	527
8	205	330	535

7. From the data given in Problem 6 compute the value of food per person for each of the various sizes of families and exhibit the data obtained by the use of bar graphs.

8. The average number of eggs produced per hen in the United States for each month of 1939 was estimated to be as follows: January, 8.0; February, 9.7; March, 14.9; April, 17.0; May, 17.0; June, 14.7; July, 13.2; August, 11.7; September, 9.3; October, 7.4; November, 6.0; December, 6.8. Prepare a bar graph showing these data.

9. The per capita consumption of beef in the United States for the years 1925 to 1934 was as follows: 1925, 68.6 lb.; 1926, 68.9 lb.; 1927, 62.3 lb.; 1928, 55.5 lb.; 1929, 56.1 lb.; 1930, 55.2 lb.; 1931, 55.0 lb.; 1932, 52.9 lb.; 1933, 58.1 lb.; 1934, 63.7 lb.; 1935, 61.0 lb. Prepare a bar graph showing these data.

10. The annual incomes from farm marketings and Government payments for the United States for the period 1933 to 1939 are given in the following table:

INCOME FROM FARM MARKETINGS AND GOVERNMENT PAYMENTS, 1933–1939
(in millions of dollars)

Year	Cash Income from Farm Marketings	Government Payments	Cash Farm Income and Government Payments
1933	5,278	131	5,409
1934	6,273	447	6,720
1935	6,969	573	7,542
1936	8,212	287	8,499
1937	8,744	367	9,111
1938	7,590	482	8,072
1939	7,733	807	8,540

Prepare a bar graph showing this data.

Area Charts

Area charts or graphs employ areas in much the same way that lengths are used in bar graphs. A correspondence is set up between a unit of area and a common measure used in the table of data, and areas are then formed proportional to the measures being represented. The two figures most often used in exhibiting areas are the rectangle and the sector of a circle.

In an area chart consisting of rectangles, the rectangles

are usually made to have equal widths, the variations in length being proportional to the measures represented. If areas of squares are employed to represent a set of measures, the lengths of the sides of the squares are proportional to the square roots of the measures represented. Comparisons of areas of squares from their appearance are usually misleading, and therefore this type of area chart is seldom used.

The areas of sectors of the same circle are particularly well adapted to showing the relation between the measure of a total quantity of some kind and the measures of the various parts which make up the total. Since the areas of sectors of the same circle are proportional to the sizes of the central angles, circular graphs or charts are easily made by simply laying off at the center of a circle angles that are proportional to the measure represented. The size of the angle to be used for any particular measure is obtained by multiplying the ratio of the particular measure to the total of the measures by 360°. That is,

$$\text{Angle} = \frac{\text{Particular measure}}{\text{Total of measures}} \times 360°.$$

If the per cent that a particular measure is of the whole quantity represented is known, then the vertical angle of the corresponding sector is simply that per cent of 360°. The

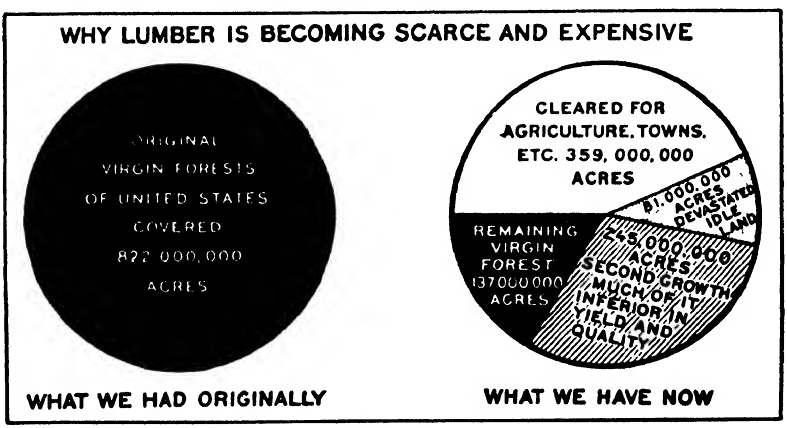


Fig. 71.

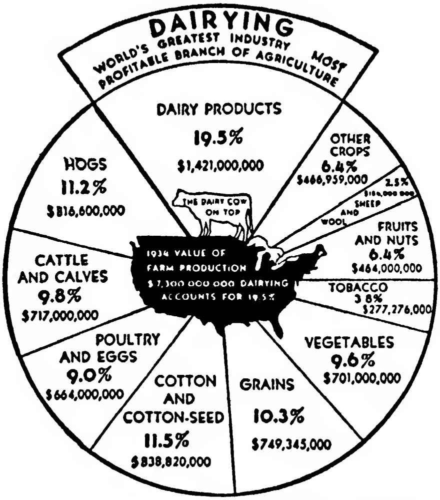
vertical angles of the sectors are easily laid off by means of a protractor.

Fig. 71 is a circular area chart which appeared in a yearbook of the United States Department of Agriculture. The angles required in dividing the circular area into sectors are shown in the accompanying table.

Distribution	Millions of Acres	Angle of Sector
Original virgin forests...........	822	360°
Cleared for agriculture, towns, etc....	359	157.2°
Second growth......	245	107.3°
Virgin forests left....	137	60°
Devastated.........	81	35.5°

Fig. 72 is an illustration of the effective use of a circular chart to emphasize the importance of the dairy industry in the United States.

The diagram shown in Fig. 73 appeared in a United States Department of Agriculture yearbook. A similar chart might be made showing the distribution of the consumer's dollar in the purchase of any commodity concerning which statistics are available.



Courtesy De Laval Separator Co.

Fig. 72.

Problems

1. The number of milk cows on farms in the South Central States in 1934 was as follows: Kentucky, 541,000; Tennessee, 513,000; Alabama, 412,000; Mississippi, 525,000; Arkansas, 438,000; Louisiana, 270,000; Oklahoma, 735,000; Texas, 1,335,000. Prepare a rectangular area chart showing these data.

2. The per cent of the carcass weight in various cuts of beef is as follows: ribs, 9.5%; loin, 18%; round, 24%; chuck, 22%; plate,

14.5%; flank and shank, 9%; kidney suet, 3%. Prepare a circular area chart showing these data.

3. The percentage composition of potatoes is as follows: water, 75%; starch, 18%; sugar, 3%; proteins, 2%; fat, 0.2%; salts, 0.7%; waste material, 1.1%. Prepare a circular area chart showing these data.

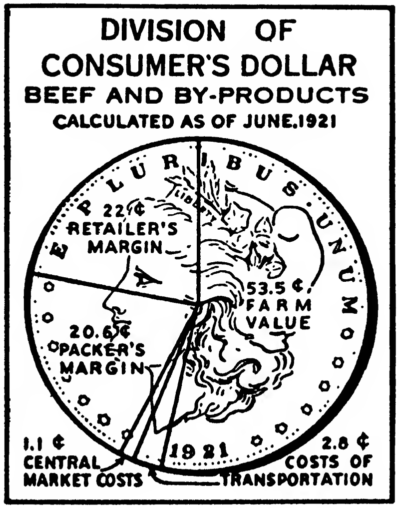


Fig. 73.

4. The per cent of the carcass weight in various cuts of pork is as follows: loin, 14%; shoulder, 9%; shoulder butts, 7%; spareribs, 2%; ham, 21%; bacon, 15%; lard, 12%; neck and feet, 4%; trimmings, 13%; waste, 3%. Prepare an area chart showing these data.

5. Fig. 74 shows the relative value of certain field crops in the United States in a certain year. Determine approximately the per cent that the value of each crop was of the value of all crops, using a protractor to measure the angles of sectors.

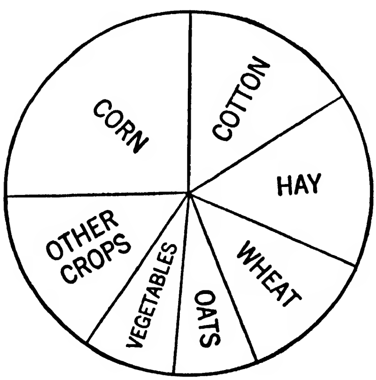


Fig. 74.

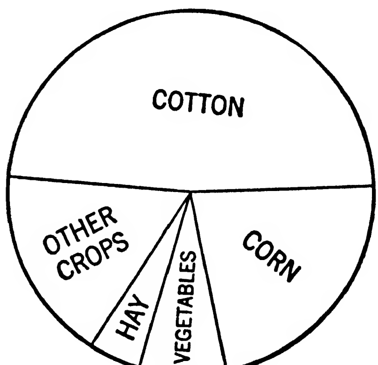


Fig. 75.

6. Fig. 75 shows the relative value of crops of the Cotton Belt in a certain year. Determine approximately the per cent that the value of each crop was of the total value.

7. The percentage composition of a fresh-laid egg is as follows: shell, 11.4%; other ash, 0.8%; protein, 13.2%; fat, 8.9%; water, 65.7%. Prepare an area chart showing these data.

8. The percentage composition of wheat grain is as follows: water, 10.5%; ash, 1.8%; protein, 11.9%; crude fiber, 1.8%; nitrogen-free extract, 71.9%; fat, 2.1%. Prepare an area chart showing these data.

9. The per cent of the carcass of a sheep in various cuts is as follows: legs, 31%; shoulder, 16%; loin and short rack, 30%; stew, 18%; waste, 5%. Prepare an area chart showing these data.

10. The acreages seeded to wheat in 1939 in the four main wheat-producing regions of the United States were estimated as follows: hard winter wheat region, 28,306,000 A.; spring wheat region, 17,034,000 A.; soft red winter wheat region, 11,965,000 A.; Pacific Northwest, 3,741,000 A. Exhibit these data by use of rectangular area charts, first with rectangles of equal length and then with squares. On which of these two types of rectangular area charts are comparisons more easily made?

11. It has been estimated that the total land area of 1,903,000,000 A. in the United States is distributed about as follows: improved land, 503,000,000 A.; forest (including cut-over and burnt-over land), 465,000,000 A.; unimproved pasture and range land, 863,000,000 A.; nonagricultural land, 72,000,000 A. Exhibit these data (a) by use of one large rectangle divided into appropriate parts, and (b) by use of circular sectors of a circle. Which of the two charts is the more readily interpreted?

12. It is estimated that about 40% of all the corn produced in the United States is fed to hogs on farms, 20% is fed to horses and mules on farms, 15% is fed to cattle on farms, 4% is fed to poultry, 3.5% is used for human food on farms, 5.5% is fed to livestock not on farms, 6.5% is ground in merchant flour mills, 1.5% is exported, and 4% is used in various other ways. Prepare a circular area chart showing these data.

13. In 1935, according to the United States Department of Agriculture Yearbook for 1940, there were 6,812,350 farm families in the United States. Of this number, 2,865,155 were farm tenants, and 716,000 of these tenants were sharecroppers. Exhibit these data by means of three rectangular areas.

14. In 1938 about 21.1% of the corn exports from the United States went to the United Kingdom, 10.6% went to Germany,

39.4% went to Canada, 13.6% went to the Netherlands, 2.1% went to Denmark, and 13.2% went to other foreign countries. Prepare an area chart showing these data.

15. A certain family's annual income is expended about as follows: rent, \$480; food, \$600; transportation, \$360; clothing, \$270; insurance and other savings, \$315; recreation, \$120; care of health, \$90; other expenditures, \$165. Show the distribution of these expenditures by use of sectors of a circle.

16. The weights of the wholesale cuts of an open side of beef weighing 291 lb. are about as follows: round and rump, 62 lb.; full loin, 65 lb.; rib, 30 lb.; chuck, 58 lb.; flank, $9\frac{1}{2}$ lb.; plate, $20\frac{1}{4}$ lb.; brisket, $21\frac{3}{4}$ lb.; fore shank, $22\frac{3}{4}$ lb.; loss in cutting, $1\frac{3}{4}$ lb. Exhibit these data by use of circular sectors.

Rectangular Coördinate Graphs

The *rectangular coördinate graph,* or *line graph,* as it is often called, usually consists of a line (straight or curved) drawn through certain points whose positions are determined by the measures given in a set of data.

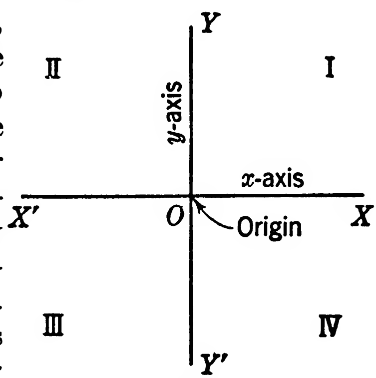


Fig. 76.

In locating points in a plane, distances and directions are considered with reference to two guide lines, called *axes,* one drawn vertically and the other drawn horizontally. The vertical reference line is usually called the y *axis,* and the horizontal line is called the x *axis.* Their point of intersection is called the *origin.* The four sections into which these lines divide the plane are called *quadrants* and are numbered I, II, III, IV, as indicated in Fig. 76. Measures of distances toward the right from the y axis are indicated by positive numbers, and measures of distances toward the left are indicated by negative numbers. Similarly, measures of distances upward from the x axis are considered positive, and measures downward are considered negative.

The position of a point in the plane is definitely determined by a pair of numbers. One number, called the *abscissa*, indicates the distance and direction of the point from the y axis; the other number, called the *ordinate*, indicates the distance and direction of the point from the x axis. Together, the *abscissa* and the *ordinate* are called the *coördinates* of the point. In indicating the coördinates of a point on a graph it is customary to write the numbers representing their values in parentheses near the point, always placing the abscissa first and separating it from the ordinate by a comma. For convenience in measuring distances, each axis may be provided with a scale, or, better still, the drawings may be made on regular coördinate paper (paper that has been cross-ruled into squares).

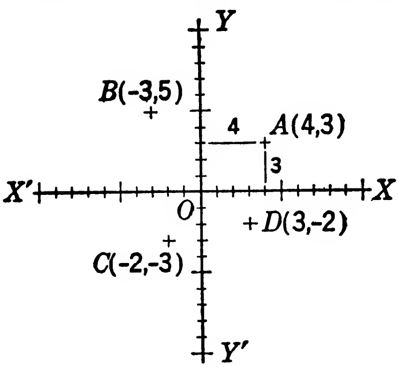


Fig. 77.

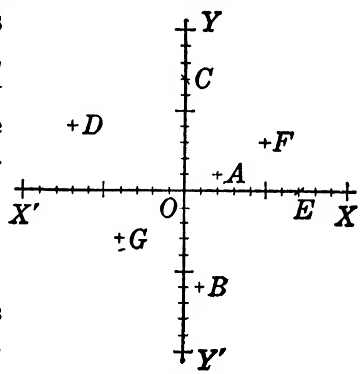


Fig. 78.

Example

In Fig. 77 are plotted the points $A(4, 3)$, $B(-3, 5)$, $C(-2, -3)$, and $D(3, -2)$.

Problems

1. Give the coördinates of each of the points A, B, C, D, E, F, and G in Fig. 78.

2. Draw a pair of axes and plot the points $P_1(-5, 2)$, $P_2(4, -3)$, $P_3(5, 2)$, and $P_4(-4, -3)$.

3. Plot the points $(0, 2)$, $(-3, 3)$, $(4\frac{1}{2}, 2)$, $(5, \frac{13}{14})$, and $(\frac{11}{3}, 0)$.

4. Draw the line that includes every point whose abscissa is 3.

5. Draw the line that includes every point whose ordinate is $5\frac{1}{2}$.

The line graph is well suited for showing a relation between two variable quantities, particularly if the measures of one of the quantities are dependent upon the measures of the other quantity. For example, the yield of wheat per acre depends upon the precipitation of moisture (by rain or snow). The relation between these two quantities, as determined approximately for a certain year's crop, is shown in the accompanying table.

Precipitation (inches)	Yield per Acre (bushels)
10	10
15	14
20	16
25	18
30	19
35	18
40	16
45	$13\frac{1}{2}$
50	$10\frac{1}{2}$
55	7

This data may be presented graphically by letting each measure of precipitation and the corresponding measure of yield be the abscissa and the ordinate, respectively, of a point, and joining from left to right by a smooth curve the points plotted. The graph obtained is shown in Fig. 79.

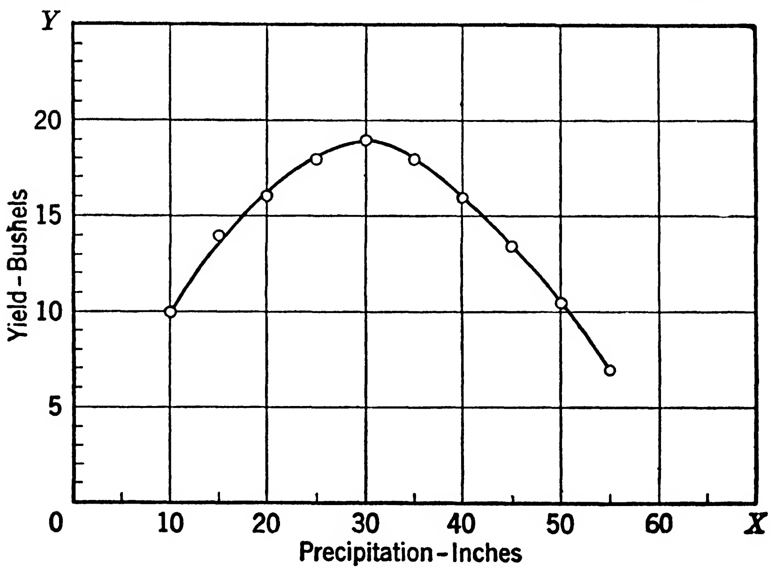


Fig. 79.

The scales used are indicated on the two axes. Notice how the ordinates (representing yield) increase as the precipitation increases, up to the point at which the precipitation is 30 in., and then begin to decrease.

Practically all of the coördinate graphs with which the student of agriculture will be concerned will lie in the first

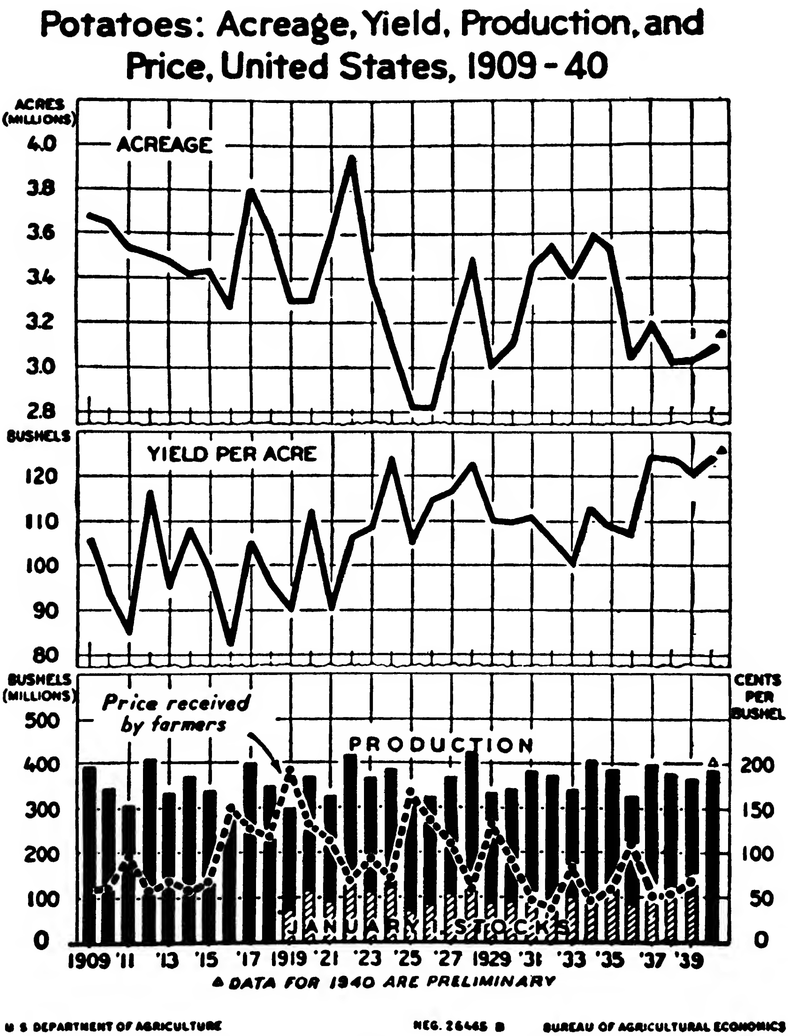


Fig. 80.

quadrant, where both the abscissa and the ordinate of every point are positive numbers.

Fig. 80 shows several related line graphs and a combination bar and line graph that were made from the data in the table below. The following brief interpretation of these

ACREAGE, YIELD, PRODUCTION, AND PRICE OF POTATOES, 1909–1940

Crop Year	Acreage (thousands of acres)	Yield per Acre (bushels)	Production (thousands of bushels)	Stocks (millions of bushels)	Price per Bushel Received by Farmers
1909	3,675	106.2	390,166		57.6¢
1910	3,644	93.9	342,052		58.4
1911	3,532	85.7	302,713		94.6
1912	3,505	115.9	406,215		56.6
1913	3,477	95.6	332,447		67.8
1914	3,417	107.8	368,249		56.2
1915	3,433	98.1	336,760		67.4
1916	3,274	82.6	270,388		149.7
1917	3,801	104.9	398,653		127.9
1918	3,597	96.2	346.114		118.8
1919	3,300	90.1	297,341	70.0	190.9
1920	3,301	111.8	368,904	112.0	132.8
1921	3,598	90.4	325,312	88.4	112.8
1922	3,901	106.5	415,373	136.7	68.5
1923	3,378	108.5	366,356	109.5	91.4
1924	3,106	123.7	384,166	120.4	71.2
1925	2,810	105.5	296,466	66.3	165.8
1926	2,811	114.4	321,607	80.4	136.1
1927	3,182	116.2	369,644	104.1	108.5
1928	3,499	122.1	427,249	130.0	57.1
1929	3,019	110.0	332,204	82.9	131.8
1930	3,103	109.8	340,572	88.4	91.9
1931	3,467	110.8	384,125	108.2	46.3
1932	3,549	106.1	376,425	109.3	39.2
1933	3,412	100.3	342,306	98.4	82.1
1934	3,597	112.9	406,105	123.7	44.8
1935	3,541	109.1	386,380	106.1	59.7
1936	3,063	108.4	331,918	85.4	114.0
1937	3,185	124.1	395,294	113.2	52.8
1938	3,023	123.8	374,163	103.6	54.8
1939	3,027	120.3	364,016	103.3	68.9
1940*	3,087	124.1	383,172		

* Preliminary.

graphs was published in the 1941 Agricultural Outlook Chart on Fruits and Vegetables:

Depth (feet)	Capacity (barrels)
8	149.2
9	167.9
10	186.5
11	205.1
12	223.8
13	242.4
14	261.1
15	279.8
16	298.4
17	317.0
18	335.7

The total acreage of potatoes in the United States was reduced sharply in 1936, and it has remained at a comparatively low level for the past 5 years. This reduction in acreage has been about offset by an increase in yields, however, and the total production of potatoes has remained as large as in most years since 1909. Prices received by farmers for potatoes tend to vary inversely with changes in production.

Problems

1. The capacities of tanks 10 ft. in diameter and of various depths are shown in the table above. Prepare a rectangular coördinate graph showing these data.

2. The temperature as read each hour from 6 A.M. to 5 P.M. of a certain winter day was as follows: 6 A.M., −12°; 7 A.M., −10°; 8 A.M. −7°; 9 A.M., −5°; 10 A.M., 0°; 11 A.M., 4°; 12 M., 8°; 1 P.M., 9°; 2 P.M., 10°; 3 P.M., 6°; 4 P.M., 0°; 5 P.M., 6°. Prepare a rectangular coördinate graph showing these data. (*Suggestion:* Represent time along the horizontal axis and temperature along the vertical axis.)

3. Determine the interest on $100 at 8% for periods of 1 to 10 yr. and prepare a rectangular coördinate graph showing the data. From the graph determine the interest on $100 for 4 yr. 3 mo.

4. The price of cotton per pound over a period of years is shown in the table at the right. Prepare a rectangular coördinate graph showing these data.

Year	Average of 10 Markets
1915	11.72¢
1916	18.96
1917	29.02
1918	29.76
1919	38.34
1920	16.66
1921	18.09
1922	25.83
1923	30.14
1924	24.22
1925	19.68
1926	14.40
1927	19.72
1928	18.67
1929	15.79
1930	9.61
1931	5.89
1932	7.15
1933	10.81
1934	12.36
1935	11.55
1936	12.70
1937	8.66
1938	8.70

5. The approximate number of tons of corn silage in a silo 12 ft. in diameter with silage at various depths is shown in the accompanying table. Prepare a rectangular coördinate graph showing these data.

Depth (feet)	Contents (tons)
10	19.8
12	24.2
14	28.7
16	33.2
18	37.8
20	42.4
22	47.0
24	51.7
26	56.5
28	61.3
30	66.1
32	70.9
34	75.8
36	80.7
38	85.5
40	90.4

6. The farm population of the United States each year from 1925 to 1940 is shown in the table below. Prepare a rectangular coördinate graph showing these data.

Year	Population (thousands)
1925	30,830
1926	30,619
1927	30,170
1928	30,188
1929	30,220
1930	30,169
1931	30,497
1932	30,971
1933	31,693
1934	31,770
1935	31,801
1936	31,809
1937	31,729
1938	31,819
1939	32,059
1940	32,245

7. The average price per 100 lb., by months, paid at Chicago in 1939 for beef steers (good grade), was as follows: January, $10.35; February, $10.23; March, $10.64; April, $10.33; May, $9.92; June, $9.29; July, $9.26; August, $9.03; September, $10.20; October, $9.68; November, $9.52; December, $9.44. Prepare a rectangular coördinate graph showing these data.

8. The average price per dozen received by farmers for eggs on the fifteenth of each month for the years 1925 to 1939 is shown in the table on the following page. (a) Make a line graph showing the variation in price from month to month for the year assigned to you by the instructor. (b) Make a chart showing the variation in the weighted average price from year to year for the period 1925 to 1939.

9. Find the lengths (in rods) of fencing required to inclose square tracts of land containing 1 A., 2 A., 3 A., 4 A., 5 A., 10 A., 15 A., 20 A., and 25 A., and prepare a rectangular coördinate graph showing the relation between the size of the tract and the length of fencing required.

10. The production of sweet potatoes and the average price

Average Price Per Dozen Received by Farmers for Eggs on the Fifteenth of Each Month for the Years 1925 to 1939

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Weighted Average
Average 1929–1938	24.2¢	20.3¢	17.3¢	16.8¢	16.8¢	16.8¢	18.1¢	19.9¢	23.2¢	26.2¢	30.1¢	28.8¢	20.3¢
1925	48.6	35.7	23.9	24.2	24.8	26.1	27.9	30.0	31.1	37.7	46.8	48.1	30.4
1926	36.3	28.9	24.1	24.8	25.2	25.7	25.7	26.4	31.5	36.8	44.9	47.6	28.9
1927	36.9	29.0	20.8	20.3	19.8	17.8	20.7	23.4	29.4	35.6	41.6	43.3	25.1
1928	38.2	29.1	23.4	22.8	24.2	23.9	25.6	27.4	31.4	34.9	39.6	42.9	28.1
1929	33.0	31.9	28.0	23.0	24.4	26.1	27.2	29.8	33.0	38.4	44.2	45.8	29.8
1930	38.4	31.8	21.3	21.5	20.0	18.6	18.8	20.6	25.3	26.5	31.7	26.8	23.7
1931	22.1	14.1	17.0	16.2	13.3	14.1	14.8	17.3	19.1	22.7	26.4	25.6	17.6
1932	17.2	12.8	10.4	10.2	10.3	10.6	12.0	14.7	17.2	22.5	26.1	28.1	14.2
1933	21.4	11.0	10.1	10.3	11.8	10.1	13.1	13.3	16.3	20.8	24.0	21.6	13.8
1934	17.6	15.8	14.4	13.5	13.3	13.2	14.1	17.2	21.9	23.7	28.6	27.0	17.1
1935	25.0	25.6	18.6	20.0	21.4	21.0	21.7	22.7	26.4	27.9	30.1	28.7	23.4
1936	22.8	23.8	17.5	16.8	18.1	18.9	20.0	22.4	24.5	27.6	32.5	30.5	21.8
1937	23.1	20.1	19.9	20.1	17.9	17.6	19.4	20.4	22.9	25.2	28.0	26.0	21.3
1938	21.6	16.4	16.2	15.9	17.6	18.2	19.9	21.0	24.9	27.1	29.0	27.9	20.3
1939	18.8	16.7	16.0	15.5	15.2	14.9	16.5	17.5	20.6	22.9	25.8	20.5	17.5

Year	Production (thousands of bushels)	Price
1925	50,241	165.1
1926	63,300	117.4
1927	70,897	109.0
1928	59,178	118.0
1929	64,963	117.1
1930	54,415	108.2
1931	66,849	72.7
1932	86,436	54.2
1933	75,248	69.5
1934	77,482	79.8
1935	83,128	70.4
1936	64,144	94.0
1937	75,053	82.5
1938	76,647	73.3
1939	72,679	75.6

per bushel received by farmers in the United States for the years 1925 to 1939 are shown in the accompanying table. Prepare a rectangular coördinate graph showing the production data by one curve and the price data by another.

11. Compute the capacities (in gallons) of cylindrical tanks 6 ft. tall and having diameters of 2 ft., 3 ft., 4 ft., 5 ft., 6 ft., 7 ft., and 8 ft., and make a rectangular coördinate graph showing the relation between the capacity of the tank and the diameter.

12. Determine the quantities of salt that should be added to 100 oz. of water to form solutions testing 2%, 4%, 6%, 8%, 10%, 12%, 14%, 16%, 18%, and 20% salt, and prepare a rectangular chart showing these data.

Graph of an Equation

In many cases the relation between two quantities may be expressed by an equation which involves letters that represent the measures of the quantities considered. By assigning numerical values to one of the letters and finding corresponding values of the other, a table of data can be set up from which a rectangular coördinate graph may be drawn. If the graph so obtained contains all points, and only those points, whose coördinates satisfy the given equation, it is called the *graph of the equation.*

As an example we may consider the relation between distance and time in the case of a body moving at a constant rate. Suppose that an automobile is traveling at the rate of 1 mi. per minute. Then if y stands for the number of miles traveled in x min., we have the relation $y = x$.

Obviously, the graph of this equation consists of every point whose y coördinate is the same as its x coördinate. The coördinates of a few such points are given in the accompanying table, and these points are plotted on the graph in Fig. 81. Many other pairs of numbers could be found that would serve as the coördinates of points on the graph.

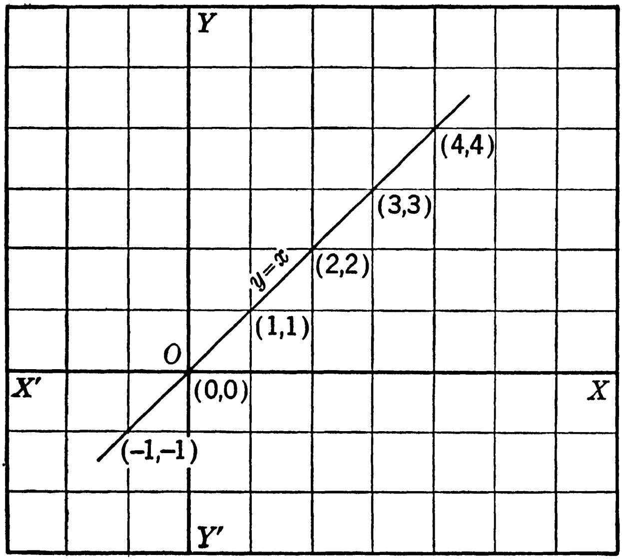


Fig. 81.

x	y
-1	-1
0	0
1	1
3	3
4	4

Next consider the relation between x and y that is expressed by the equation $x + y = 5$. To obtain a point on the graph of this equation one needs merely to find a pair of numbers, one a value of x and the other the corresponding value of y, whose sum is 5. A brief table of values is shown at the right. The points represented by these pairs of numbers are plotted in Fig. 82.

x	y
-1	6
0	5
1	4
2	3
3	2
$3\frac{1}{2}$	$1\frac{1}{2}$
5	0

In making a table of values for the graph of

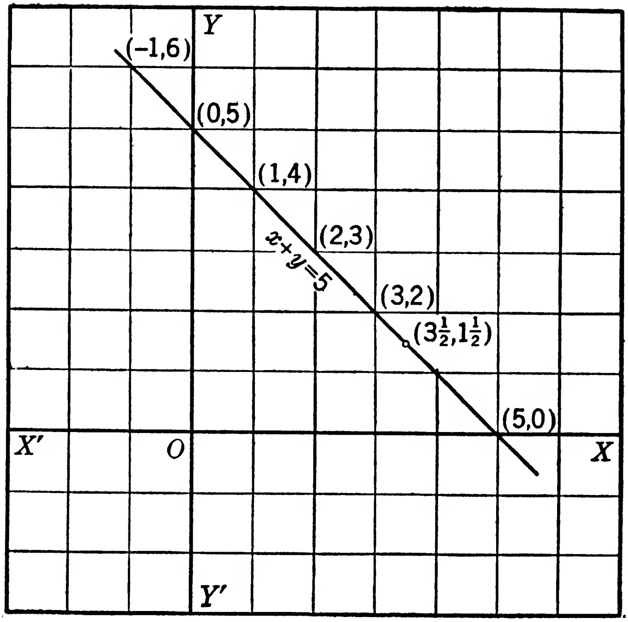


Fig. 82.

an equation, it is usually convenient to solve the equation for one letter in terms of the other. For example, suppose the equation under consideration is

$$2x + 3y = 12.$$

Solving this equation for y, we obtain

$$y = \frac{12 - 2x}{3}.$$

x	y
-2	$\frac{16}{3}$
0	4
1	$\frac{10}{3}$
2	$\frac{8}{3}$
3	2
6	0
7	$-\frac{2}{3}$

Now we may proceed to assign values to x and find corresponding values of y, thus forming a table of paired numbers like the one at the right. The points represented by these pairs of numbers are plotted on the graph in Fig. 83.

A complete treatment of the subject of equations and their graphs, which is one of the main topics in the field of analytic geometry, is beyond the scope of this book.

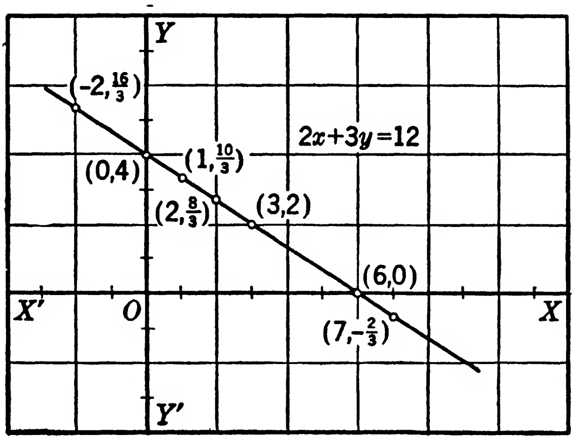


Fig. 83.

Problems

1. Write an equation indicating the relation between the interest on \$100 at 6% per annum and the number of years this sum is at interest. Make a table of values and draw the graph of the equation.

2. If r stands for the number of linear units in the length of the radius of a circle and A stands for the number of square units in the area of the circle, the relation between A and r is expressed by the equation $A = \pi r^2$. Draw the graph of this equation.

Draw the graph of each of the following equations:

3. $y = 3x$.

4. $y = 3x + 2$. Compare this graph with that of the preceding problem.

5. $3y = 2x$.

6. $6y = 4x$. Compare this graph with that of the preceding problem. Note that the two equations express the same relation between x and y.

7. $3x - 2y = 6$.

8. $y = x^2$.

9. $y = x^2 - 2x - 15$.

10. $y = x^3$

CHAPTER 9

Special Applications of Practical Measurements

Concrete Making

Concrete is made by mixing cement, sand, and gravel in various proportions with enough water to give the mixture a workable consistency. The proportion in which the dry ingredients are mixed depends upon the use to be made of the finished concrete. By a 1-2-4 mixture is meant a mixture consisting by volume of 1 part of cement, 2 parts of sand, and 4 parts of gravel. In high-grade concrete the sand with its voids or air spaces fills the voids in the gravel and the cement fills the remaining voids, so that the volume of the resulting mixture is much less than the total volume of the ingredients used. In computing the materials needed for concrete work either of two methods may be used with satisfactory results.

Methods of Computing Volumes of Materials Needed

Method I. This method is based upon the fact that about 42 cu. ft. of unmixed cement, sand, and gravel are required to make 1 cu. yd. (27 cu. ft.) of concrete. Since $42 \div 27 = 1\frac{5}{9}$, the sum of the volumes of unmixed materials required is about $1\frac{5}{9}$ times the volume of concrete to be made.

Example

How many cubic feet each of cement, sand, and gravel are required to make 240 cu. ft. of a 1-2-4 mixture of concrete?

$1\frac{5}{9} \times 240 = 373\frac{1}{3}$, the number of cubic feet of unmixed materials required.

In a 1-2-4 mixture the volume of cement is one-seventh of the total volume of materials.

$\frac{1}{7} \times 373\frac{1}{3} = 53\frac{1}{3}$, the number of cubic feet of cement.
$2 \times 53\frac{1}{3} = 106\frac{2}{3}$, the number of cubic feet of sand.
$4 \times 53\frac{1}{3} = 213\frac{1}{3}$, the number of cubic feet of gravel.

In practice, one might assume that 53 cu. ft. of cement, 107 cu. ft. of sand, and 213 cu. ft. of gravel are required.

Method II. Voids are usually considered as making up 45% of a volume of gravel or crushed stone and $33\frac{1}{3}$% of a volume of sand. Therefore about 55% of the volume of gravel used, $66\frac{2}{3}$% of the volume of sand used, and all of the cement used actually go toward making the volume of finished concrete. The per cents of void spaces in the sand and gravel can be used as a basis for computing the quantities of materials needed to produce a certain quantity of concrete, as indicated in the following example.

Example

Using the per cents of void spaces mentioned above, determine the number of cubic feet each of cement, sand, and gravel required to make 240 cu. ft. of a 1-2-4 mixture of concrete.

Let C = the number of cubic feet of cement required,
S = the number of cubic feet of sand required,
and G = the number of cubic feet of gravel required.

Then, since all of the volume of the cement, $66\frac{2}{3}$% of the volume of the sand, and 55% of the volume of the gravel together make up the required volume of concrete, we have

$$C + \tfrac{2}{3}S + .55G = 240.$$

Since a 1-2-4 mixture is to be used, it follows that

$$S = 2C$$

and

$$G = 4C.$$

The system of equations

$$\begin{cases} (1) \quad C + \tfrac{2}{3}S + .55G = 240 \\ (2) \quad S = 2C \\ (3) \quad G = 4C \end{cases}$$

is readily solved by using Equations (2) and (3) to substitute for S and G in Equation (1).

$$
\begin{aligned}
C + \tfrac{2}{3}(2C) + .55(4C) &= 240. \\
C + \tfrac{4}{3}C + 2.20C &= 240. \\
3C + 4C + 6.6C &= 720. \\
13.6C &= 720. \\
C &= 52.9, \text{ approximately.} \\
S &= 2(52.9) = 105.8, \text{ approximately.} \\
G &= 4(52.9) = 211.6, \text{ approximately.}
\end{aligned}
$$

Thus, in a 1-2-4 mixture about 53 cu. ft. of cement, 106 cu. ft. of sand, and 212 cu. ft. of gravel are required to make 240 cu. ft. of concrete. Note that these results are consistent with those obtained by Method I.

Other methods. Several other methods of estimating the quantities of materials required in concrete making are used to some extent.

One method, often referred to as "the 10% method" or as "the 90% method," is based upon the assumption that the volume of gravel required is about 90% of the volume of concrete to be made. In practice, the volume of gravel required is often arrived at by subtracting from the volume of concrete 10% of itself. Estimates are easily formed on this basis but are not considered as reliable as those obtained by Methods I and II.

Applying this method to the problem previously solved by Methods I and II, we obtain

$$
\begin{aligned}
10\% \text{ of } 240 &= 24, \\
240 - 24 &= 216, \text{ number of cubic feet of gravel,} \\
\tfrac{1}{4} \text{ of } 216 &= 54, \text{ number of cubic feet of cement,}
\end{aligned}
$$

and

$$
2(54) = 108, \text{ number of cubic feet of sand.}
$$

As a basis for making quickly a rough estimate of the quantities of materials needed, one may assume that the total volume of unmixed materials required is about $1\frac{1}{2}$ times the volume of concrete to be made. Using this assumption on the problem previously considered, we have

$$
\begin{aligned}
1\tfrac{1}{2} \times 240 &= 360, \text{ number of cubic feet of materials,} \\
360 \div 7 &= 51, \text{ number of cubic feet of cement,} \\
51 \times 2 &= 102, \text{ number of cubic feet of sand,}
\end{aligned}
$$

and

$$51 \times 4 = 204, \text{ number of cubic feet of gravel.}$$

The student may find this procedure of value in checking results obtained by the more reliable methods. He can at least tell in this way whether or not the results obtained are reasonable.

Formulas for Computing Volumes of Materials Needed

The assumption that 42 cu. ft. of unmixed materials are required to make 1 cu. yd. of concrete is used as the basis for the following formulas, which the student may readily verify. If C, S, and G denote the numbers of cubic feet of cement, sand, and gravel, respectively, required to make 1 cu. yd. of concrete in which the number of parts of cement, sand, and gravel is c, g, and s, respectively, then

$$C = \frac{42c}{c + s + g},$$

$$S = \frac{42s}{c + s + g},$$

and

$$G = \frac{42g}{c + s + g}.$$

Usually, c is taken as 1 in a ratio formula; however, it is sometimes convenient to replace a 1-2½-4 formula, for example, by the equivalent 2-5-8 formula.

Proportions of cement, sand, and gravel commonly used in concrete work are given in Table IV, page 168.

Problems

By Method I find the amount of materials needed per cubic yard of the following concrete mixtures:

1. A 1-2-5 mixture.

2. A 1-1-2 mixture.

3. A 1-1½-3 mixture.

4. A 1-3-6 mixture.

5. A $1\text{-}2\frac{1}{2}\text{-}5$ mixture.

6-10. Solve Problems 1-5 by use of Method II, using the per cents of void spaces in sand and gravel mentioned above.

11. Compute the quantity of materials needed in making 200 cu. ft. of a 1-2-3 concrete.

12. How much cement, sand, and gravel are needed to construct a concrete walk 4 ft. wide, 30 ft. long, and 6 in. thick if a 1-2-5 mixture is used?

13. What quantities of ingredients are required for making 40 cylindrical concrete pillars 20 in. in diameter and 30 in. long if a 1-2-4 mixture is used?

14. How much cement, sand, and gravel are needed to make 25 concrete piers each 2 ft. long and each having a lower base 2 ft. square and an upper base 1 ft. square if a 1-2-4 mixture is used?

15. What amounts of materials are needed in constructing a wall 1 ft. thick for a cylindrical concrete silo 40 ft. high with an inside diameter of 12 ft. if the mixture is $1\text{-}2\frac{1}{2}\text{-}4$?

16. How much cement, sand, and gravel are required to construct a concrete floor 40 ft. long, 30 ft. wide, and 6 in. thick if a 1-3-5 mixture is used?

17. How many cubic feet each of cement, sand, and gravel are required in making 35 concrete cylindrical pillars 2 ft. in diameter and 30 in. long if a $1\text{-}2\frac{1}{2}\text{-}5$ mixture is used?

18. A concrete watering trough 6 ft. long, 3 ft. wide, and 2 ft. deep (outside measurements) with walls and bottom 6 in. thick is made from a 1-2-3 mixture. If the sand and gravel are bought already mixed at $2 per cubic yard and the cement is bought at 80¢ per cubic foot, what is the total cost of cement, sand, and gravel used in making the trough?

19. Calculate the raw materials needed in a $1\text{-}2\frac{1}{2}\text{-}3$ mixture to build a sidewalk 4 ft. wide, 90 ft. long, and 5 in. thick. If cement is 80¢ a sack and sand and gravel are $2 a cubic yard, what would be the cost of these materials?

20. A concrete foundation for a house 24 ft. by 36 ft. (outside measurements) is to be 12 in. wide at the bottom, 8 in. wide at the top, and 28 in. deep. How many cubic feet each of cement, sand, and gravel are needed if a 1-2-4 mixture is used?

21. How many cubic feet of cement, sand containing 30% voids, and gravel containing 50% voids must be used in a 1-2-4 mixture to make 1000 cu. ft. of concrete?

22. How much cement, sand containing 35% voids, and gravel containing 40% voids are needed in a 1-2½-5 mixture to make 1000 cu. ft. of concrete?

Measurement of Lumber

The measure of lumber is expressed in board feet (bd. ft.), 1 bd. ft. being defined as the measure of a piece of lumber 1 ft. square and 1 in. thick. The number of board feet in a piece of lumber is given by the formula

$$\text{Bd. ft.} = \frac{\text{Thickness in inches} \times \text{Width in inches} \times \text{Length in feet}}{12},$$

where thicknesses of less than 1 in. are counted as 1 in. and other thicknesses are counted as they are given in the description of the piece of lumber.

Problems

Find the number of board feet in each of the following:

1. 32 planks 3″ × 8″ × 16′.

2. 50 pieces 2″ × 4″ × 12′.

3. 12 pieces 4″ × 6″ × 14′.

4. 200 pieces ½″ × 1½″ × 16′.

5. 150 pieces ¾″ × 4″ × 24′.

6. At $40 per M (1000 bd. ft.), how much should the siding (1 in. thick) for a barn 28 ft. by 56 ft. cost if the wall is 10 ft. high and no allowance is made for openings?

7. About how many board feet of sheathing 1 in. by 4 in. are required for a gable roof of a barn if each side of the roof measures 18 ft. by 40 ft. and the laths are placed 4 in. apart?

8. Allowing for 3 pickets ¾ in. by 2 in. by 4 ft. for each linear foot of fence, find the number of board feet of lumber in a 100-ft. roll of pickets.

9. The walls, gables, and roof sheathing of a garage 20 ft. long and 18 ft. wide with walls 8 ft. high are made of lumber 1 in.

thick. The roof has a pitch of $\frac{1}{2}$, and there is no overhang. Find the number of board feet of this kind of lumber required, making no allowance for openings.

10. The floor of a building measures 28 ft. by 60 ft. The sills measure 4 in. by 6 in. in cross section and extend along the sides and ends and lengthwise through the center; the floor joists measure 2 in. by 8 in. by 14 ft., rest on the sills, crosswise to the building, and are 2 ft. apart from center to center. Find the number of board feet of lumber in the sills and floor joists of this building.

11. The wall of a cylindrical silo 14 ft. in outside diameter and 35 ft. high is constructed of wooden staves 2 in. thick. Determine approximately the number of board feet of lumber contained in the wall of this silo.

12. From frequent practice in computing board measure, lumbermen associate with the various descriptions of cross sections of lumber certain numbers that may be applied directly to total lengths to give board measure. The number applied in a particular case is one-twelfth of the number of square inches in the cross-sectional area of a piece of the lumber. For example, the number of board feet in a lot of 2 in. by 4 in. pieces of lumber is simply $(2 \times 4)/12$, or $\frac{2}{3}$, times the total number of linear feet of the lumber. Complete the table at the right, in which the board measure of certain sizes of lumber is expressed in terms of the length.

Description of Cross Section (inches)	Board Feet in Terms of Length
1×2	....
1×4	$\frac{1}{3}$ of length
1×6	....
1×8	....
1×12	....
2×4	....
2×6	....
2×8	....
2×12	....
4×4	....
4×6	....
6×6	....
6×8	....
8×10	....
8×12	....

Measuring Lumber in the Log

For practical purposes the number of board feet of lumber that may be obtained from a log of certain size is given

with fair accuracy by the following rule, which is known as *Doyle's Rule:*

From the number of inches in the smallest diameter of the log subtract 4, multiply the remainder by half itself, multiply this result by the number of feet in the length of the log, and divide by 8.

For example, the number of board feet in a log 16 ft. long and having a smallest diameter of 18 in. is approximately

$$\frac{(18 - 4) \times 7 \times 16}{8} = \frac{14 \times 7 \times 16}{8} = 196.$$

Another formulation of the above rule is

$$\text{Bd. ft.} = \frac{(\text{Least diameter in inches} - 4)^2 \times \text{Length in feet}}{16}.$$

Problems

Determine approximately the number of board feet of lumber in logs with the following dimensions:

1. Length, 24 ft.; smallest diameter, 12 in.
2. Length, 20 ft.; smallest diameter, 14 in.
3. Length, 28 ft.; smallest diameter, 30 in.
4. Length, 16 ft.; smallest diameter, 24 in.
5. Length, 18 ft.; smallest diameter, 18 in.

Measuring Lumber in the Tree

Table V, page 168, gives approximately the number of board feet of lumber in trees of various diameters and consisting of one, two, or three 16-ft. logs.

Measuring Hay in Stacks

In all problems thus far considered involving the measurement of hay in stacks we have assumed the stack of hay in each case to be in the form of some geometric solid whose volume could be readily computed. The student realizes that most haystacks are not of such ideal shape. Then how

may the content of a haystack be measured satisfactorily?

As a result of a study made by the United States Department of Agriculture in coöperation with a number of state agricultural experiment stations, in which several thousand oblong and round stacks were measured in the Western and Great Plain States, the following methods for volume measurement were developed.*

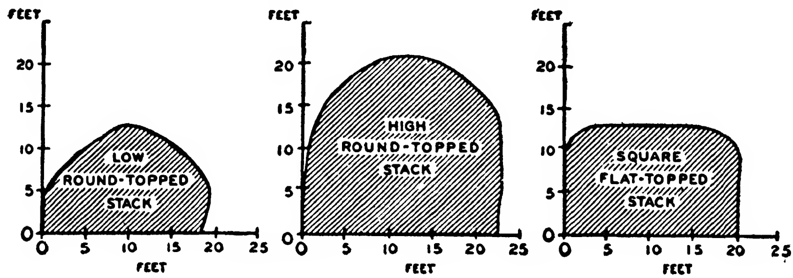


Fig. 84.

Oblong Stacks

All oblong stacks are divided into three types, as indicated in Fig. 84. The volume (denoted by V) in each case is computed from three measurements: the width of the stack at the ground (denoted by W), the average length of the stack (denoted by L), and the "over" (denoted by T), which is the distance from the ground on one side over the stack to the ground on the other side. The formulas for making the computations are as follows:

For low round-topped stacks,

$$V = (.52T - .44W)WL;$$

for high round-topped stacks,

$$V = (.52T - .46W)WL;$$

and for square flat-topped stacks,

$$V = (.56T - .55W)WL.$$

* Leaflet No. 72, United States Department of Agriculture.

Example

Find the number of cubic feet of hay in a high round-topped stack 20 ft. wide, 45 ft. over, and 50 ft. long.

$$\begin{aligned} V &= [.52(45) - .46(20)](20)(50) \\ &= [23.40 - 9.20](1000) \\ &= [14.20](1000) \\ &= 14{,}200. \end{aligned}$$

The stack contains approximately 14,200 cu. ft.

Round Stacks

A fairly satisfactory formula for the volume, V, of a round stack is given in terms of the circumference, C, of the base and the over, T, taken over the highest point:

$$V = (.04T - .012C)(C^2).$$

Leaflet No. 72, United States Department of Agriculture, also gives a useful table of volumes of round stacks whose circumferences lie between 45 and 98 ft. and whose overs lie between 25 and 50 ft.

Example

Determine the volume of a round stack 100 ft. in circumference and having an over of 60 ft.

$$\begin{aligned} V &= (.04 \times 60 - .012 \times 100)(100)^2 \\ &= 12{,}000. \end{aligned}$$

The volume is approximately 12,000 cu. ft.

In actual practice the diagrams in Fig. 84 are useful in determining whether an oblong stack is high round-topped or low round-topped.

Problems

Find approximately the volume of each of the stacks of hay described below:

1. Low round-topped, 18 ft. wide, 35 ft. over, 40 ft. long.
2. Square flat-topped, 20 ft. wide, 45 ft. over, 50 ft. long.

3. High round-topped, 25 ft. wide, 55 ft. over, 56 ft. long.
4. Round, 80 ft. in circumference, 45 ft. over.
5. Round, 100 ft. in circumference, 60 ft. over.

Measurements Relating to Silos

In most sections of the country, after feed stored in a silo has begun to be used, a cross-sectional slice of certain minimum thickness, depending mainly upon the kind of silage and surrounding climatic conditions, must be fed at regular intervals to prevent spoilage of the silage. On account of this fact the maximum cross-sectional area of a silo should, in most cases, be determined by the least quantity of silage that is to be fed daily over any portion of the feeding period.

Example

What should be the dimensions of a cylindrical silo of maximum cross-sectional area which is to provide each of 30 cows with 40 lb. of silage daily for 140 da. if it is desirable to remove a slice of silage 3 in. thick each day to prevent spoilage?

First, the diameter of the silo should be determined.

Let r = the number of feet in the radius of the silo.

1 cu. ft. of silage weighs about 40 lb. Then the slice to be fed daily contains about 30 cu. ft.

$$3 \text{ in.} = \tfrac{1}{4} \text{ ft.}$$

Then

$$\pi r^2(\tfrac{1}{4}) = 30.$$

$$\pi r^2 = 120.$$

$$r^2 = \frac{120}{\pi} = 38.2, \text{ approximately.}$$

$$r = 6.2, \text{ approximately.}$$

The silo should be about 12.4 ft. in inside diameter.

The number of feet in the height of the silo is simply the number of days in the feeding period times the number of feet in the thickness of the slice to be removed each day. Thus,

$$140 \times \tfrac{1}{4} = 35.$$

The silo should be about 35 ft. high.

Problems

1. A certain cylindrical silo is 40 ft. high and 14 ft. in inside diameter. (a) If 1 cu. ft. of silage weighs 40 lb., how many tons of silage can be stored in this silo? (b) How many pounds of silage would be contained in a cross-sectional slice 3 in. thick? (c) Such a slice would provide how many cows with 40 lb. of silage each? (d) If a 3-in. slice is fed daily, the silo provides for a feeding period of how long?

2. What should be the dimensions of a cylindrical silo for a herd of 36 cows if a ration of 40 lb. is fed daily for 120 da. and it is desirable to remove a 4-in. slice of silage each day?

3. A silo is 16 ft. in inside diameter and 36 ft. high. If it is desirable to feed a 3-in. slice daily, this silo will provide a daily ration of 30 lb. per head for how many cows? For what length of feeding period does the silo provide?

4. What should be the dimensions of a cylindrical silo to provide 30 cows with 40 lb. each daily for 100 da., 30 lb. each for 50 da., and 20 lb. each for 30 da., if the minimum thickness of slice to be removed daily over any portion of the feeding period is 2.5 in.? (If a given set of requirements indicate that the height of a silo should be greater than 50 ft., it is usually considered best to build two silos of suitable heights, having together the required capacity, instead of one.)

5. A rule often followed in determining the proper cross-sectional area of a silo is the allowance of a cross-sectional feeding surface of 5 sq. ft. for each cow to be fed. If the daily ration is 1 cu. ft. of silage per head, application of the above rule would result in removal of a slice of silage of what thickness each day?

6. The cross section of a certain silo is a hexagon (a plane figure with six sides) each side of which is 6 ft. long (inside measurement). Determine the cross-sectional area of the silo and from that the minimum number of cattle that should be fed 1 cu. ft. each per day to insure a feeding depth of 2 in. per day. If this silo is 35 ft. high, about how many tons of silage does it hold when it is full?

7. The wall of a certain silo consists of 12 sides each 4 ft. wide (inside measurement). If the silo is 40 ft. high, about how many tons of silage will it hold when full?

8. A certain trench silo having a trapezoidal cross section measuring 7 ft. across the bottom, 11 ft. across the top, and 8 ft. in depth, is 50 ft. long. If it is desirable in feeding to remove a

slice 4 in. thick crosswise to the trench each day, what is the minimum number of cattle that should be fed 1 cu. ft. of silage per day? On this basis, for what length of feeding period does the silo provide? What is the capacity of the silo in tons if 1 cu. ft. weighs 30 lb.?

9. A trench silo is to be constructed so as to provide 1 cu. ft. of silage per head daily for a herd of 25 cattle. If a slice 4 in. thick is to be fed daily, what should be the cross-sectional area of the trench? Work out a set of dimensions for the silo, making it 10 ft. deep and long enough to provide for a 90-da. feeding period.

Feeds

The treatment of feeds given here is not intended to be considered authoritative or complete. The aim is simply to familiarize the student with certain calculations applicable to problems that arise in a study of feeds.

As previously stated, the main constituents of feeds are (1) proteins, (2) ether extract (fats and oils), (3) nitrogen-free extract (chiefly carbohydrates, such as sugars and starches), (4) crude fiber (cell walls and woody materials of plants), (5) ash (residue obtained from burning a feed), and (6) water. The first four of these are usually spoken of as the nutrients of a feed. Table VII, page 170, gives the average percentage composition of certain feeding stuffs. The digestibility of feed nutrients varies with different feeds containing them and with different animals consuming them. However, from numerous feeding experiments average figures, called *coefficients of digestibility,* have been obtained which indicate for a particular feed the per cents of the nutrients that are digestible. The numbers of pounds of digestible nutrients per 100 lb. of certain feeds, as indicated in Table VIII, page 171, were obtained by applying such a set of digestibility coefficients to the weights of the nutrients in 100 lb. shown in Table VII.

Table VIII also gives for each feed listed the number of therms of productive energy furnished by 100 lb. of the feed. A therm is the amount of heat or energy required to raise

the temperature of 1000 kg of water 1° C. While figures on productive energy are usually obtained from experimental measurement of the fat-producing quality of feeds, they may be considered as indicative of the value of feeds for animal maintenance and for production of flesh, animal products such as milk and eggs, and energy to be used in performing work.

A factor usually considered significant in determining whether or not a feed or feed mixture constitutes a balanced ration for an animal is the *nutritive ratio* of the ration. The nutritive ratio of a feed or ration is the ratio of the weight of digestible crude protein to the weight of digestible carbohydrate equivalents furnished by the nonprotein nutrients contained in the feed or ration. The weight of digestible carbohydrate equivalents is the sum of the weights of digestible nitrogen-free extract, digestible crude fiber, and 2.25 times the weight of digestible ether extract. The weight of digestible ether extract (mainly fats) is multiplied by 2.25 for the reason that as a source of energy 1 lb. of fat is equivalent to about 2.25 lb. of carbohydrate. In expressing a nutritive ratio the first term (or numerator) of the ratio is taken as 1, and the second term (or denominator) is computed by combining the weight of digestible nitrogen-free extract and digestible crude fiber with 2.25 times the weight of digestible ether extract and dividing this sum by the weight of digestible crude protein. Since the totals of digestible nutrients given in Table VIII include 2.25 times the digestible fats, the second term of the ratio for a feed may be obtained by subtracting the weight of digestible protein from the total weight of digestible nutrients and dividing this difference by the weight of digestible protein. Thus, if R denotes the nutritive ratio, T denotes the total weight of digestible nutrients, and P denotes the weight of digestible protein, we have

$$R = \frac{1}{(T - P) \div P}.$$

In balanced rations for different types of livestock, nutritive ratios usually vary within the following ranges:

Beef cattle	1 : 4 to 1 : 9
Dairy cattle	1 : 3 to 1 : 7
Hogs	1 : 4 to 1 : 7
Horses	1 : 6 to 1 : 9
Poultry	1 : 3 to 1 : 8
Sheep	1 : 5 to 1 : 9

Various standards of feeding requirements for different types of livestock have been derived from feeding experiments. It is not within the scope of this book to discuss the merits of these standards or to examine the bases upon which they are founded. Table IX, page 172, gives a set of standards based upon animal requirements of (1) productive energy, (2) digestible crude protein, and (3) bulk of dry matter in ration.

Costs of feeds may be compared on the basis of content of one or more of the following: digestible protein, total digestible nutrients, productive energy.

Examples

1. Determine the nutritive ratio of a horse and mule feed consisting of 50 lb. of corn, 35 lb. of oats, 14 lb. of wheat bran, and 1 lb. of salt.

	Digestible Protein	*Total Digestible Nutrients*
Corn	$50 \times .064 = 3.200$	$50 \times .848 = 42.400$
Oats	$35 \times .096 = 3.360$	$35 \times .719 = 25.165$
Wheat bran	$14 \times .133 = 1.862$	$14 \times .568 = 7.952$
Total	8.422	75.517

Digestible nonprotein $= 75.517 - 8.422 = 67.095$.

$$67.095 \div 8.4 = 7.97, \text{ approximately.}$$

Hence, 1 : 7.97 is the nutritive ratio of this feed mixture.

2. Compute the number of therms of productive energy furnished by 10 lb. of hog feed consisting of 6 lb. of corn meal, 1.5 lb. of ground oats, 1.5 lb. of wheat gray shorts, .5 lb. of tankage, and .5 lb. of cottonseed meal.

$.06 \times 86.8 = 5.2080$, number of therms furnished by corn meal.
$.015 \times 71.9 = 1.0785$, number of therms furnished by oats.
$.015 \times 75.7 = 1.1355$, number of therms furnished by shorts.
$.005 \times 59.7 = .2985$, number of therms furnished by tankage.
$.005 \times 74.9 = .3745$, number of therms furnished by cottonseed meal.

8.0950, or 8.1, approximate number of therms of productive energy supplied by 10 lb. of this feed.

3. Determine the number of pounds each of corn, cottonseed meal, and cottonseed hulls needed to form a ration whose bulk is about 24 lb. and which is to supply 1.6 lb. of digestible protein and 13 therms of productive energy.

It is assumed that the 24 lb. of bulk includes the normal amount of water contained in the several feeds and that reduction to a basis of dry matter content is not necessary in this case.

The method used here may be described as that of successive trial replacements.

First, suppose that the roughage, cottonseed hulls, makes up the total bulk of 24 lb. This quantity of hulls provides $24 \times .179 = 4.296$ therms of productive energy.

Suppose now that some of the hulls are replaced (pound for pound) by sufficient corn to increase the productive energy from 4.296 to the required 13 therms. For each pound of hulls replaced by a pound of corn a gain of $.848 - .179 = .669$ therms occurs. Hence $(13 - 4.296) \div .669 = 8.704 \div .669 = 13$ is the number of pounds of corn to be used in this first replacement.

The number of pounds of digestible protein called for in the ration is now compared with that contained in the 11 lb. of hulls and 13 lb. of corn. These quantities of hulls and corn contain $11 \times .004 + 13 \times .064 = .044 + .832 = .876$ lb. of digestible protein. Cottonseed meal, which is high in protein, may now replace some of the corn in sufficient quantity to raise the protein content from .876 lb. to the desired 1.6 lb. For each pound of corn replaced by a pound of cottonseed meal a gain of $.359 - .064 = .295$ lb. of digestible protein occurs. Hence $(1.6 - .876) \div .295 = .724 \div .295 = 2.45$ lb. of cottonseed meal are used in this replacement.

This replacement of 2.45 lb. of corn by an equal quantity of cottonseed meal lowers the productive energy content by $2.45 (.848 - .749) = 2.45 \times .099 = .24$ therms. Addition of .3 lb. of corn restores this amount of productive energy and does not seriously affect the bulk nor the digestible protein content.

The quantities of the specified feeds required for this ration are about as follows:

Feed		*Pounds*
Cottonseed hulls	$24 - 13 =$	11.0
Corn	$13 - 2.45 + .3 =$	10.85
Cottonseed meal		2.45
Total bulk		24.30

This example may be solved by forming and solving a system of equations. If x, y, and z denote, respectively, the number of pounds each of hulls, corn, and cottonseed meal, we obtain the following three equations:

$$
\begin{aligned}
&(1)\quad x + y + z = 24.\\
&(2)\quad .004x + .064y + .359z = 1.6.\\
&(3)\quad .179x + .848y + .749z = 13.
\end{aligned}
$$

Equations (1), (2), and (3) are based upon the ration requirements of bulk, digestible protein, and productive energy, respectively. The results obtained by solving this system differ little from those obtained by the trial replacement method.

In formulating a ration on the basis of bulk, protein, and energy requirements, only one combination of three specified feeds is possible; but, if more than three feeds are to be used in the mixture, many satisfactory combinations can usually be worked out.

4. On the basis of content of digestible protein only, (a) what would be a fair price per bushel to pay for oats if corn is worth 60¢ per bushel? (b) Compare the value of 100 lb. of oats with the value of 100 lb. of corn.

(a) 1 bu. of corn contains $56 \times .064 = 3.584$ lb. of digestible protein.

1 bu. of oats contains $32 \times .096 = 3.072$ lb. of digestible protein.

A bushel of oats then should be worth $\frac{3.072}{3.584} \times 60$¢, or 51.4¢.

(b) 100 lb. of corn contains 6.4 lb. of digestible protein, while 100 lb. of oats contains 9.6 lb. of digestible protein. Then oats should be worth 9.6/6.4, or 1.5, times as much per 100 lb. as corn. If corn is worth 60¢ per bushel, it is worth about $1.07 per hundredweight, and oats should be worth about $1.60 per hundredweight.

Problems

1. Determine the nutritive ratio of each of these feeds: (a) corn, (b) oats, (c) alfalfa leaf meal, (d) cottonseed meal.

2. On the basis of total digestible nutrients only, compare the values of equal weights of corn and oats. If corn is worth 80¢ per bushel, what should oats be worth per hundredweight?

3. How many pounds of corn are required to provide as much digestible protein as is contained in 100 lb. of cottonseed meal?

4. How many pounds of Sudan grass hay provide as much productive energy as that supplied by 100 lb. of ground oats?

5. How many pounds each of corn and cottonseed meal are required for the two together to provide digestible protein and productive energy equivalent to that provided by 100 lb. of wheat bran?

6. Compute the nutritive ratio of a horse and mule feed consisting of the following: corn, 70 lb.; oats, 15 lb.; cottonseed meal, 7 lb.; molasses, 7 lb.; and salt, 1 lb.

7. Commercial mixed feeds usually bear tags indicating analyses guaranteed by manufacturers. Such an analysis shows approximately the per cents of crude protein, crude fat, crude fiber, and nitrogen-free extract present in the feed. Work out an analysis of this kind for the following poultry-fattening mixture: corn, 34 lb.; wheat gray shorts, 20 lb.; kafir chops, 20 lb.; dried buttermilk, 15 lb.; ground oats, 10 lb.; and salt, 1 lb.

8. Determine the nutritive ratio of a dairy cow feed consisting of the following: cottonseed meal, 20 lb.; corn meal, 33 lb.; wheat bran, 15 lb.; ground whole oats, 12 lb.; molasses, 12 lb.; alfalfa leaf meal, 5 lb.; ground limestone, 2 lb.; and salt, 1 lb.

9. Compute the number of therms of productive energy supplied by the 100 lb. of feed described in Problem 8. If a dairy cow is fed 3 lb. of this mixture for each gallon of milk she produces, how many therms of productive energy are provided per day by this feed for a cow that is producing 4 gal. of milk daily?

10. Determine the nutritive ratio of a hog feed consisting of 88 lb. of corn meal and 6 lb. each of tankage and cottonseed meal.

11. Find approximately the number of pounds each of corn, cottonseed meal, and cottonseed hulls required to form a ration whose bulk is about 20 lb. and which provides 1.5 lb. of digestible protein and 12 therms of productive energy.

12. Compute the nutritive ratio of the following poultry scratch feed: corn, 40 lb.; kafir, 30 lb.; wheat, 20 lb.; and barley, 10 lb.

13. Formulate a suitable ration for a 500-lb. growing (and fattening) steer, basing the requirements on the standards given in Table IX and selecting feeds from those listed in Table VIII.

14. Work out a suitable ration for a 1200-lb. horse at medium work, referring to Tables VIII and IX for needed data.

15. Work out a suitable ration for a 900-lb. dairy cow which is producing 4 gal. of 5% milk daily.

16. Suppose that 100 lb. of live-weight pork can be produced from 700 lb. of shelled corn or from a mixture consisting of 300 lb. of shelled corn, 25 lb. of tankage, and 25 lb. of cottonseed meal. If shelled corn is worth 80¢ per bushel, and cottonseed meal is worth $1.50 per hundredweight, how much could a feeder afford to pay for tankage so that the cost of pork production from feeding the mixture would not exceed the cost from feeding corn alone?

17. How many pounds of cottonseed supply as much digestible protein as is contained in 100 lb. of cottonseed meal?

18. How many pounds each of corn and cottonseed meal fed together are equivalent to 100 lb. of ground oats in the amounts of digestible protein and productive energy provided?

19. As sources of digestible protein only, compare the costs of tankage at $70 per ton and cottonseed meal at $36 per ton.

20. As a source of productive energy only, what should wheat gray shorts be worth per hundredweight when corn is selling at 90¢ per bushel?

Fertilizers

As previously stated, the main plant foods added to soil by applications of fertilizers are nitrogen, phosphoric acid (P_2O_5), and potash. A prepared fertilizer is usually described by indicating in per cents its content of these three ingredients. For example, a fertilizer is properly designated as a "4-8-6 fertilizer" if 4% of it by weight is nitrogen, 8% is available phosphoric acid, and 6% is potash.

The following examples will illustrate methods of solving certain types of problems included in this chapter. In problems pertaining to valuations of fertilizers, the monetary worth of a fertilizer will be considered as identical with the total value of its plant food content.

Examples

1. Determine the three largest whole numbers proportional to 1, 3, and 1, representing, respectively, the per cents of nitrogen, phosphoric acid, and potash, obtainable for a fertilizer formed from cyanamid testing 22% nitrogen, superphosphate testing 18% phosphoric acid, and muriate of potash testing 50% potash.

It is convenient to consider data regarding 100 lb. of fertilizer.

Let x = the number of pounds of nitrogen in 100 lb. of this fertilizer.

Then $\frac{x}{.22}$ = the number of pounds of cyanamid required to furnish the x lb. of nitrogen,

$3x$ = the number of pounds of phosphoric acid in 100 lb. of this fertilizer,

and $\frac{3x}{.18}$ = the number of pounds of superphosphate required to furnish the $3x$ lb. of phosphoric acid.

Also x = the number of pounds of potash in 100 lb. of the fertilizer,

and $\frac{x}{.50}$ = the number of pounds of muriate of potash required to furnish the x lb. of potash.

Now, since as little filler as possible is to be used, the sum

$$\frac{x}{.22} + \frac{3x}{.18} + \frac{x}{.50}$$

is to be as near 100 as is possible with x an integer. If

$$\frac{x}{.22} + \frac{3x}{.18} + \frac{x}{.50} = 100,$$

then

$$\frac{x}{22} + \frac{x}{6} + \frac{x}{50} = 1,$$

or

$$75x + 275x + 33x = 1650.$$

Adding, we obtain

$$383x = 1650.$$

Therefore

$$x = 4.3, \text{ approximately.}$$

But x is an integer. Therefore $x = 4$ and $3x = 12$. Hence a 4-12-4 fertilizer can be formed from the specified materials. The student may show that about 7 lb. of each 100 lb. of this fertilizer must be filler.

2. If nitrogen is worth 12¢ per pound, available phosphoric acid is worth 6¢ per pound, and potash is worth 5¢ per pound, what should be the value of 1 T. of 6-8-4 fertilizer?

$$
\begin{aligned}
6\% \text{ of } 2000 &= 120, \text{ number of pounds of nitrogen.}\\
120 \times \$0.12 &= \$14.40, \text{ value of nitrogen.}\\
8\% \text{ of } 2000 &= 160, \text{ number of pounds of phosphoric acid.}\\
160 \times \$0.06 &= \$9.60, \text{ value of phosphoric acid.}\\
4\% \text{ of } 2000 &= 80, \text{ number of pounds of potash.}\\
80 \times \$0.05 &= \$4, \text{ value of potash.}\\
\$14.40 + \$9.60 + \$4 &= \$28, \text{ value of 1 T. of this fertilizer.}
\end{aligned}
$$

3. Suppose that on the basis of costs of unmixed fertilizer materials it is estimated that nitrogen is worth 10¢ per pound, available phosphoric acid is worth 6¢ per pound, and potash is worth 4¢ per pound. Assuming that the costs per pound of the three plant foods present in prepared fertilizers are proportional, respectively, to these values, determine the price paid for each of the three plant foods in a 4-8-6 fertilizer retailing at $36 per ton.

Let x = the number of cents paid per pound of nitrogen,
y = the number of cents paid per pound of phosphoric acid,
and z = the number of cents paid per pound of potash.

$$
\begin{aligned}
4\% \text{ of } 2000 &= 80, \text{ number of pounds of nitrogen in 1 T.}\\
8\% \text{ of } 2000 &= 160, \text{ number of pounds of phosphoric acid in 1 T.}\\
6\% \text{ of } 2000 &= 120, \text{ number of pounds of potash in 1 T.}
\end{aligned}
$$

Then

$$80x + 160y + 120z = 3600,$$

or

$$(1)\quad 2x + 4y + 3z = 90.$$

Also,

$$\frac{x}{y} = \frac{10}{6},$$

or

$$(2)\qquad y = \frac{3x}{5},$$

and

$$\frac{x}{z} = \frac{10}{4},$$

or

$$(3)\qquad z = \frac{2x}{5}.$$

Using (2) and (3) to substitute for y and z in (1), we obtain

$$2x + \frac{12x}{5} + \frac{6x}{5} = 90,$$

or

$$10x + 12x + 6x = 450.$$

Adding, we obtain

$$28x = 450.$$

Therefore

$$x = 16.07, \text{ approximately.}$$

From (2) $$y = \frac{3(16.07)}{5} = \frac{48.21}{5} = 9.64, \text{ approximately.}$$

From (3) $$z = \frac{2(16.07)}{5} = \frac{32.14}{5} = 6.43, \text{ approximately.}$$

The prices paid per pound of plant food, then, are approximately 16.1¢ for nitrogen, 9.6¢ for available phosphoric acid, and 6.4¢ for potash.

Problems

1. How many pounds each of nitrate of soda, superphosphate (18%), and muriate of potash are required in making a ton of 6-8-4 fertilizer? How much filler is needed?

2. Determine the per cent of each of the three main plant foods present in a prepared fertilizer consisting of 840 lb. of superphosphate (18%), 1000 lb. of cottonseed meal, and 160 lb. of muriate of potash.

3. How many pounds each of superphosphate (20%) and cottonseed meal are required to form a ton of fertilizer testing 6% nitrogen and 8% available phosphoric acid? What per cent of the mixture is potash?

4. If nitrogen, available phosphoric acid, and potash in fertilizers are considered worth, respectively, 14¢, 7¢, and 6¢ per pound, what should be the cost of 4-8-4 fertilizer to be applied to 60 A. of corn land at the rate of 250 lb. per acre?

5. If 400 lb. of a 6-8-4 fertilizer are mixed with 600 lb. of a 4-12-4 fertilizer, what are the per cents of the three plant foods in the mixture?

6. Determine the three largest whole numbers, proportional to 1, 2, and 1 and indicating, respectively, the per cents of nitrogen, phosphoric acid, and potash, obtainable for a fertilizer to be formed from nitrate of soda, superphosphate (20%), and muriate of potash.

7. In comparing valuations of fertilizers in 1939–1940 the Texas Agricultural Experiment Station used the following approximate average costs per pound of plant food: 12.0¢ for nitrogen, 6.5¢ for available phosphoric acid, and 6.0¢ for potash. At these prices what would be a fair price per ton to pay for 6-10-7 fertilizer?

8. If the prices paid per pound of nitrogen, available phosphoric acid, and potash are proportional, respectively, to the average costs given in Problem 7, what price is paid per pound for each of the plant foods if $36 is paid for a ton of 4-10-4 fertilizer?

9. How much 4-8-4 fertilizer contains the same amount of each of the plant foods as a ton of 6-12-6 fertilizer? If 6-12-6 fertilizer is worth $36 per ton, what should 4-8-4 fertilizer be considered worth?

10. Is it more economical to buy 4-12-4 fertilizer at $32 per ton or 5-15-5 fertilizer at $38 per ton?

11. If 800 lb. of cottonseed meal are added to a ton of 3-10-3 fertilizer, what per cent of each of the plant foods is contained in the resulting mixture?

12. Determine the per cent of each of the plant foods in a sack of fertilizer containing 25 lb. of nitrate of soda, 67 lb. of superphosphate (20%), and 8 lb. of muriate of potash.

13. Determine the three largest whole numbers, proportional to 1-3-1 and indicating, respectively, the per cents of nitrogen, phosphoric acid, and potash, obtainable for a fertilizer to be formed from sulphate of ammonia, superphosphate (20%), and sulphate of potash.

14. At the average costs per pound of plant food given in Problem 7, what should be the value of a fertilizer consisting of 800 lb. of superphosphate (18%), 800 lb. of cottonseed meal, 200 lb. of nitrate of soda, and 200 lb. of muriate of potash?

15. Compute the quantity of each of the following plant food carriers required in making a ton of 4-10-4 fertilizer: sulphate of ammonia furnishing the nitrogen, superphosphate (20%) supplying the phosphoric acid, and kainit and muriate of potash together furnishing the potash in such quantities that no filler is required.

Mixtures

Formulas

If a person is engaged in work that requires the solving of a great number of mixture problems of the same kind, he saves time by using a formula instead of setting up and solving a system of equations for each problem. The formula is derived from a system of equations for solving a general type of mixture problem.

Consider the problem of finding the number of pounds each of milk testing $r_1\%$ butterfat and milk testing $r_2\%$ butterfat required to make N lb. of milk testing $r\%$ butterfat. Without loss of generality we may assume that r_2 is greater than r_1.

Let N_1 = the number of pounds of milk testing $r_1\%$ required,

and N_2 = the number of pounds of milk testing $r_2\%$ required.

We may now write the following system of equations:

$$(1)\quad N_1 + N_2 = N.$$

$$(2)\quad r_1N_1 + r_2N_2 = rN.$$

Multiplying each member of Equation (1) by r_2 and subtracting the corresponding members of Equation (2), we have

$$\begin{aligned} r_2N_1 + r_2N_2 &= r_2N. \\ r_1N_1 + r_2N_2 &= rN. \\ \hline (r_2 - r_1)N_1 &= (r_2 - r)N. \end{aligned}$$

$$(a)\quad N_1 = \frac{r_2 - r}{r_2 - r_1}N.$$

In like manner, we may eliminate N_1 and find

$$(b)\quad N_2 = \frac{r - r_1}{r_2 - r_1}N.$$

Formulas (a) and (b) express the values of N_1 and N_2 in terms of numbers that are known, and the solution of a particular problem of this type is obtained at once by making proper substitutions for letters in the right-hand side of each formula.

The Mixture Lever

If each member of Formula (a) is divided by the corresponding member of Formula (b), the result is the proportion

$$\frac{N_1}{N_2} = \frac{r_2 - r}{r - r_1}.$$

Various devices are used to help the student remember this proportion or to suggest to him certain manipulations involved in its application. One such device is the "*mixture lever*," so called because it employs a principle analogous to that of the balanced lever.

The proportion

$$\frac{N_1}{N_2} = \frac{r_2 - r}{r - r_1}$$

may be associated with a lever in the following way. On a straight-line segment representing a lever, the per cent r of butterfat in the final mixture is placed at a point corresponding to the fulcrum of the lever; and the numbers of pounds, N_1 and N_2, of the component parts of the mixture are then placed on the line segment at points whose distances from the fulcrum are such that the per cent differences $r - r_1$ and $r_2 - r$ may be interpreted as the lever arms of N_1 and N_2, respectively. See Fig. 85.

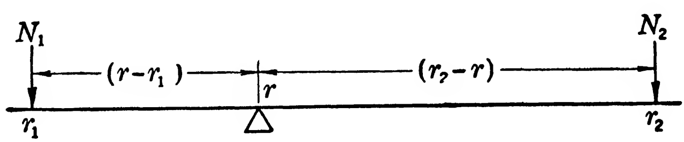


Fig. 85.

The equation

$$\frac{N_1}{N_2} = \frac{r_2 - r}{r - r_1}$$

then conforms to the principle of the lever.

Written in the form

$$N_1(r - r_1) + N_2(r - r_2) = 0,$$

this equation may be extended as a formula to cover the case in which several quantities of milk testing different per cents of butterfat are mixed. If m such quantities, N_1 lb., N_2 lb., N_3 lb., . . . , N_m lb., of milk testing $r_1\%$, $r_2\%$, $r_3\%$, . . . , $r_m\%$ of butterfat, respectively, are mixed, we have

$$N_1(r - r_1) + N_2(r - r_2) + N_3(r - r_3) + \cdots + N_m(r - r_m) = 0.$$

In a particular problem the value of any one of the letters involved in this equation may be found, provided that the values of all of the other letters are known.

Example

A mixture is formed of 50 lb. of milk testing 6% butterfat, 80 lb. testing 4.5%, 100 lb. testing 4%, and 60 lb. testing 3.5%. The mixture should contain what per cent of butterfat?

Let r = the per cent of butterfat in the mixture.

Then

$$50(r - 6) + 80(r - 4.5) + 100(r - 4) + 60(r - 3.5) = 0,$$

or

$$50r - 300 + 80r - 3600 + 100r - 400 + 60r - 2100 = 0.$$

Collecting terms, we obtain

$$290r = 1270.$$

Therefore

$$r = 4.4,$$

approximately.

The mixture contains approximately 4.4% butterfat.

Problems

1. A mixture formed from 80 lb. of milk testing 5.2% butterfat and 120 lb. testing 3.5% butterfat should contain what per cent butterfat?

2. How many pounds of milk testing 6% butterfat should be added to the mixture mentioned in Problem 1 to raise the butterfat content to 5%?

3. How many pounds of nitrate of soda testing 16% nitrogen should be added to 500 lb. of 2-8-4 fertilizer to form a fertilizer testing 4% nitrogen?

4. Determine the per cent of protein in the following mixture for dairy cows: 400 lb. of corn meal, 200 lb. of wheat bran, and 400 lb. of cottonseed meal.

5. A milk distributor formed a mixture consisting of the following lots of milk bought from producers: 150 lb. testing 4.5% butterfat, 210 lb. testing 3% butterfat, 100 lb. testing 4% butterfat, and 80 lb. testing 5.3% butterfat. What per cent butterfat was contained in the mixture?

CHAPTER 10

Exponents; Logarithms; The Slide Rule

Exponents

Exponents play an important role in the development of certain processes and devices used to shorten the work of multiplication, division, computation of powers, and extraction of roots. For this reason, a brief review of the definitions and laws of exponents is given here.

In the following tables m and n denote positive integers and a, b, and c represent numbers different from zero.

Definitions	Examples	
1. $a^n = a \cdot a \cdot a \cdot \cdots \cdot a$ (n factors).	$a^3 = a \cdot a \cdot a$.	$5^3 = 5 \cdot 5 \cdot 5 = 125$.
2. $a^{-n} = \dfrac{1}{a^n}$.	$a^{-3} = \dfrac{1}{a^3}$.	$5^{-3} = \dfrac{1}{5^3} = \dfrac{1}{125}$.
3. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.	$a^{\frac{2}{3}} = \sqrt[3]{a^2}$.	$5^{\frac{2}{3}} = \sqrt[3]{5^2}$.
4. $a^0 = 1$.	$a^0 = 1$.	$5^0 = 1$.

Laws	Examples	
1. $a^m \cdot a^n = a^{m+n}$.	$a^3 \cdot a^2 = a^5$.	$2^3 \cdot 2^2 = 2^5 = 32$.
2. $\dfrac{a^m}{a^n} = a^{m-n}$.	$\dfrac{a^5}{a^2} = a^{5-2} = a^3$.	$\dfrac{2^5}{2^2} = 2^{5-2} = 2^3 = 8$.
3. $(a^m)^n = a^{mn}$.	$(a^2)^3 = a^6$.	$(2^2)^3 = 2^6 = 64$.
4. $(abc)^n = a^n b^n c^n$.	$(abc)^3 = a^3 b^3 c^3$.	$(3bc^2)^2 = 9b^2c^4$.
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$.	$\left(\dfrac{a}{b}\right)^4 = \dfrac{a^4}{b^4}$.	$\left(\dfrac{2}{3}\right)^4 = \dfrac{2^4}{3^4} = \dfrac{16}{81}$.

Logarithms

In the statement $2^3 = 8$, 2 is regarded as the *base*, 3 is an *exponent* (or *exponent of power*), and 2^3, or 8, is called a

power of 2. Also, the exponent 3 is called the *logarithm* of 8 with respect to the base 2 and is denoted by $\log_2 8$. In general, if $a^x = N$, x is called the logarithm of N with respect to the base a (symbolically, $\log_a N = x$).

Since a logarithm is an exponent, certain properties of logarithms are derived from the laws of exponents. These properties and their connection with the laws of exponents are exhibited in the following table. In this table, M and N denote positive numbers.

Laws of Exponents	Properties of Logarithms
I. If $M = a^x$ and $N = a^y$, then $MN = a^x a^y = a^{x+y}$.	I. $\log_a MN = \log_a M + \log_a N$.
II. If $M = a^x$ and $N = a^y$, then $\dfrac{M}{N} = \dfrac{a^x}{a^y} = a^{x-y}$.	II. $\log_a \dfrac{M}{N} = \log_a M - \log_a N$.
III. If $M = a^x$, then $M^p = (a^x)^p = a^{px}$ and $\sqrt[q]{M} = M^{\frac{1}{q}} = (a^x)^{\frac{1}{q}} = a^{\frac{x}{q}}$.	III. $\log_a M^p = p \log_a M$ and $\log_a \sqrt[q]{M} = \dfrac{1}{q} \log_a M$.

For shortening the work in arithmetical computations it is convenient to consider all positive numbers as powers of 10. By methods not within the scope of this book it is possible to determine to a required number of decimal places the logarithm of any positive number with respect to the base 10. In other words, if N denotes a positive number, a number x can be found so that $10^x = N$ within any range of approximation desired. (In this book $\log N$ means $\log_{10} N$.)

In the first column of the table on page 146 are listed certain numbers and in the second column opposite each of these numbers is given the logarithm of that number with respect to the base 10. Explanations given in the third column are intended to make clear the connection between a number and its logarithm but not to suggest a general method for obtaining logarithms.

Number	Logarithm	Explanation
.001	-3	$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = .001.$
.01	-2	$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = .01.$
.1	-1	$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = .1.$
1.	0	$10^0 = 1.$
2.1544	$\frac{1}{3}$	$10^{\frac{1}{3}} = \sqrt[3]{10} = 2.1544$, approximately.
3.1623	$\frac{1}{2}$	$10^{\frac{1}{2}} = \sqrt{10} = 3.1623$, approximately.
4.6416	$\frac{2}{3}$	$10^{\frac{2}{3}} = \sqrt[3]{10^2} = \sqrt[3]{100} = 4.6416$, approximately.
10	1	$10^1 = 10.$
31.623	$\frac{3}{2}$	$10^{\frac{3}{2}} = \sqrt{10^3} = \sqrt{1000} = 31.623$, approximately.
100	2	$10^2 = 100.$
316.23	$\frac{5}{2}$	$10^{\frac{5}{2}} = \sqrt{10^5} = \sqrt{100{,}000} = 316.23$, approximately.
1000	3	$10^3 = 1000.$
3162.3	$\frac{7}{2}$	$10^{\frac{7}{2}} = \sqrt{10^7} = \sqrt{10{,}000{,}000} = 3162.3$, approximately.
10,000	4	$10^4 = 10{,}000.$

The significant figures of a number include the first non-zero figure at the left and all the figures that follow it. If two numbers are made up of the same sequence of significant figures but differ in the position of the decimal point, then their logarithms to the base 10 differ by a whole number. Thus, $3162.3 = 100 \times 31.623$. By Property I, $\log 3162.3 = \log 100 + \log 31.623 = 2 + \log 31.623$. In the above table note that $\log 31.623 = 1.5$, and $\log 3162.3 = 3.5$. The property just mentioned makes it desirable to express a logarithm that is not a whole number as a *sum* of an *integer* (positive or negative) and a *positive decimal fraction.* The integer thus involved is called the *characteristic* of the logarithm, and the positive decimal fraction is called the *mantissa* of the logarithm. Thus the characteristic of the logarithm of 3162.3 is 3, and the mantissa is .5. Table XI, page 183, gives to four decimal places the mantissas of logarithms of numbers up to 1000. The characteristic of the logarithm of a number is readily determined by inspection. From the above table observe that numbers between 1 and 10 have logarithms between 0 and 1, and each such loga-

rithm has 0 for its characteristic; numbers between 10 and 100 have logarithms between 1 and 2, and each such logarithm has 1 as its characteristic; numbers between 100 and 1000 have logarithms between 2 and 3, and each such logarithm has 2 for its characteristic; and so on. A rule may now be stated: *If a number is greater than 1, the characteristic of its logarithm is positive, or zero, and is one less than the number of significant digits to the left of the decimal point.*

Numbers between .1 and 1 have logarithms between -1 and 0, and each such logarithm, being the sum of a *positive decimal* fraction and a *whole* number, has -1 for its characteristic; numbers between .01 and .1 have logarithms between -2 and -1, and each such logarithm has -2 for its characteristic; numbers between .001 and .01 have logarithms between -3 and -2, and each such logarithm has -3 for its characteristic; and so on. These facts suggest this rule: *If a number is expressed as a positive decimal fraction, the characteristic of its logarithm is negative and is numerically one more than the number of zeros immediately following the decimal point.*

Use of Table of Mantissas

As already pointed out, numbers consisting of the same sequence of figures have the same mantissa in their logarithms. It follows, too, that zeros contained at the extreme right in a number may be disregarded in determining a mantissa. Thus 6, 60, and 600 have the same mantissa in their logarithms, and this mantissa, .7782, is listed in Table XI in the column headed by 0 and is opposite the 60 of the N column. The mantissas of the logarithms of all one-digit and two-digit numbers are listed in the column headed by 0. The mantissa of the logarithm of a *three-digit* number is found in the column headed by the third digit of the number and is opposite the entry of the other two digits in the N column. Consult the table and check these values: $\log 3.5 = 0.5441$; $\log 35 = 1.5441$; $\log 350 = 2.5441$; $\log 356 = 2.5514$; $\log 357 = 3.5527$. Note that $\log .356 = -1 + .5514$; and, while

this could be written as $-.4486$, it is more convenient to express the -1 as $9 - 10$ and to write $\log .356 = 9.5514 - 10$. Similarly, $\log .00356 = -3 + .5514 = 7.5514 - 10$.

In determining the mantissa of the logarithm of a number consisting of more than three digits, the first and last of which are different from zero, it is convenient to place temporarily a decimal point after the first three digits and consider the number as being between two three-digit numbers listed in the table. An example will illustrate: Consider log 35.642. The characteristic of log 35.642 is 1. The mantissa is the same as that for 356.42 and is considered as .42 of the way between the mantissa of log 356 and that of 357.

$$
\begin{array}{lllll}
 & & \textit{Number} & \textit{Mantissa} & \\
\text{Increase} & & & & \\
1\left\{\begin{array}{l} \\ \\ \\ \end{array}\right. & .42\left\{\begin{array}{l} \\ \\ \end{array}\right. & \begin{array}{l} 357 \\ 356.42 \\ \\ 356 \end{array} & \left.\begin{array}{r} .5527 \\ ? \\ \\ .5514 \end{array}\right\} & \begin{array}{l} \text{Increase} \\ .0013 \\ \\ \\ \end{array}
\end{array}
$$

$$.42 \text{ of } .0013 = .000546$$

An increase of 1 in the number is accompanied by an increase of .0013 in the mantissa; then an increase of .42 in the number should be accompanied by an increase of approximately .42 of .0013, which is .000546, or about .0005. Hence the mantissa of log 356.42 is $.5514 + .0005$, or .5519, and $\log 35.642 = 1.5519$.

In finding a number whose logarithm is known, the sequence of figures making up the number is obtained by locating as nearly as possible the mantissa of the given logarithm in Table XI; the position of the decimal point is then determined by seeing that the characteristic of the given logarithm conforms to the rules previously given for characteristics.

Suppose that $\log N = 1.8993$. What number is N? In Table XI, .8993 is found to be the mantissa of the logarithm of all numbers having the sequence of figures 793. Since the characteristic of $\log N$ is 1, it follows that $N = 79.3$.

If $\log A = 2.8993$, then $A = 793$.
If $\log B = 3.8993$, then $B = 7930$.
If $\log C = 0.8993$, then $C = 7.93$.
If $\log D = 9.8993 - 10$, then $D = .793$.
If $\log E = 8.8993 - 10$, then $E = .0793$.

Next suppose that $\log M = 1.8996$. The mantissa .8996 is not given in Table XI.

$$.0005\left\{\begin{array}{l} .0003\left\{\begin{array}{l} 1.8993 = \log 79.3 \\ \\ 1.8996 = \log M \end{array}\right. \\ \\ \qquad\qquad 1.8998 = \log 79.4 \end{array}\right.\left.\vphantom{\begin{array}{l}1\\1\\1\\1\\1\end{array}}\right\}.1$$

It seems reasonable to conclude that M is between 79.3 and 79.4 and further that M is approximately three-fifths of the way from 79.3 to 79.4.

$$\tfrac{3}{5} \text{ of } .1 = .06.$$

Therefore

$$M = 79.3 + .06 = 79.36, \text{ approximately.}$$

Problems

Without the use of a table, find the value of the letter involved in each of the following:

1. $5^x = 25$.
2. $\log_5 25 = X$.
3. $\log_3 N = 4$.
4. $\log_a 8 = 3$.
5. $\log_2 32 = X$.
6. $\log_{10} N = 3$.
7. $10^x = .001$
8. $\log_{10} .001 = X$.
9. $\log_{10} N = 0$.
10. $\log_{10} 10{,}000 = X$.

By use of Table XI find each of the following:

11. log 367.
12. log 3.67.
13. log .00367.
14. log 36,700.
15. log 3674.
16. N if $\log N = 1.6839$.
17. A if $\log A = 2.9542$.
18. B if $\log B = 3.9628$.
19. N if $\log N = 8.7210 - 10$.
20. M if $\log M = 1.4752$.

Computations by Use of Logarithms

A few examples will illustrate how logarithms are useful in obtaining results of multiplication, division, and computations of powers and roots. Since most of the mantissas of logarithms given in Table XI are approximations, the results obtained by use of logarithms are usually only approximately correct. Arrangement of work in the form used here is suggested.

Examples

1. Compute (36.2)(8.4).

Let $N = (36.2)(8.4).$

Then $\log N = \log 36.2 + \log 8.4.$

$$\begin{aligned} \log 36.2 &= 1.5587. \\ \log 8.4 &= \underline{0.9243.} \end{aligned}$$

Adding, $\log N = 2.4830.$

$N = 304.1$, approximately.

2. Compute $\frac{574.8}{6.24}\cdot$

Let $N = \frac{574.8}{6.24}\cdot$

Then $\log N = \log 574.8 - \log 6.24.$

$$\begin{aligned} \log 574.8 &= 2.7595. \\ \log 6.24 &= \underline{0.7952.} \end{aligned}$$

Subtracting, $\log N = 1.9643.$

$N = 92.1$, approximately.

3. Compute $\frac{(.0258)(437)}{60.82}\cdot$

Let $N = \frac{(.0258)(437)}{60.82}\cdot$

Then $\log N = \log .0258 + \log 437 - \log 60.82.$

$$\begin{aligned} \log .0258 &= 8.4116 - 10. \\ \log 437 &= \underline{2.6405.\quad\quad} \end{aligned}$$

Adding, log numerator $= 11.0521 - 10$, or $1.0521.$

$\log 60.82 = \underline{1.7840.\quad\quad}$

Subtracting, $\log N = 9.2681 - 10.$

$N = .1854$, approximately.

4. Compute $(83.5)^3$.

$$\begin{aligned}
\text{Let} \qquad N &= (83.5)^3. \\
\text{Then} \qquad \log N &= 3 \log 83.5 \\
&= 3(1.9217) \\
&= 5.7651. \\
N &= 582{,}200, \text{ approximately.}
\end{aligned}$$

In this case the last three digits of N cannot be considered accurate. Actual multiplication gives $(83.5)^3 = 582{,}182.875$. A more extensive table of logarithms could be used to give the degree of accuracy desired.

5. (a) Compute $\sqrt[3]{32.96}$.

$$\begin{aligned}
\text{Let} \qquad N &= \sqrt[3]{32.96}. \\
\text{Then} \qquad \log N &= \tfrac{1}{3} \log 32.96 \\
&= \tfrac{1}{3}(1.5180) \\
&= .5060. \\
N &= 3.206, \text{ approximately.}
\end{aligned}$$

(b) Compute $\sqrt[3]{.3296}$.

$$\begin{aligned}
\text{Let} \qquad N &= \sqrt[3]{.3296}. \\
\text{Then} \qquad \log N &= \tfrac{1}{3} \log .3296 \\
&= \tfrac{1}{3}(9.5180 - 10) \\
&= \tfrac{1}{3}(29.5180 - 30) \\
&= 9.8393 - 10. \\
N &= .6907.
\end{aligned}$$

Note that -1 may be written as $29 - 30$ and that, in this case, this form is more convenient than the form $9 - 10$.

6. Compute $\dfrac{(83.5)(.063)^{\frac{2}{3}}}{400\sqrt{51.7}}$.

$$\text{Let} \qquad X = \frac{(83.5)(.063)^{\frac{2}{3}}}{400\sqrt{51.7}}.$$

$$\text{Then} \qquad \log X = (\log 83.5 + \tfrac{2}{3} \log .063) - (\log 400 + \tfrac{1}{2} \log 51.7).$$

$$\begin{aligned}
\log 83.5 &= && 1.9217. \\
\tfrac{2}{3} \log .063 &= \tfrac{2}{3}(8.7993 - 10) \\
&= \tfrac{1}{3}(17.5986 - 20) \\
&= \tfrac{1}{3}(27.5986 - 30) = && \underline{9.1995 - 10.}
\end{aligned}$$

$$
\begin{aligned}
&\text{Adding,} \qquad \log \text{numerator} &&= 11.1212 - 10 \\
& &&= 1.1212.
\end{aligned}
$$

$$
\begin{aligned}
\log 400 &= && 2.6021. \\
\tfrac{1}{2}\log 51.7 &= \tfrac{1}{2}(1.7135) &&= \underline{\ .8567.} \\
\text{Adding,} \quad & \log \text{denominator} &&= 3.4588.
\end{aligned}
$$

$$
\begin{aligned}
\log \text{numerator} &= 1.1212 = 11.1212 - 10. \\
\log \text{denominator} &= \qquad\quad 3.4588. \\
\text{Subtracting,} \quad & \\
\log X &= \qquad\quad 7.6624 - 10. \\
X &= .004596.
\end{aligned}
$$

Problems

Compute the following by use of logarithms:

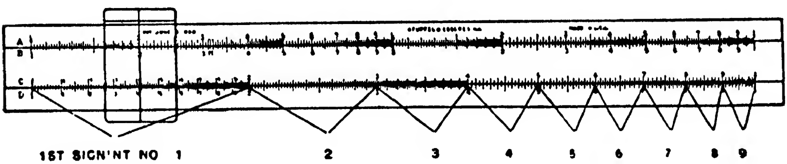
1. $(28.4)(3.95)$
2. $(741.8)(.063)$
3. $\dfrac{35.91}{8.72}$
4. $\dfrac{7.3682}{.581}$
5. $\dfrac{(83)(6.705)}{429.7}$
6. $(8.37)^3$
7. $\sqrt[5]{16.27}$
8. $(1.5)^{\frac{3}{2}}$
9. $\dfrac{305\sqrt{.87}}{(37.4)^2}$
10. $\dfrac{(47.28)(.507)^{\frac{3}{2}}}{.099\sqrt[3]{12.459}}$

The Slide Rule

As a first step toward explaining the principles and methods employed in the operation of the slide rule, it may be pointed out that results of addition and subtraction of numbers can be indicated graphically by use of two similar linear scales. Two 1-ft. rules, each graduated in inches, may be used for this purpose. For convenience, one rule may be referred to as the "M scale" and the other as the "N scale." To add 3 and 2, place the 0 of the N scale even with the 3 of the M scale and have the scales alongside each other and

extending in the same direction. The 2 of the N scale then falls even with the 5 of the M scale, thus indicating that the sum of 3 and 2 is 5. To subtract 6 from 10, place the 6 of the N scale even with the 10 of the M scale, keeping the scales alongside each other and extending in the same direction. The 0 of the N scale is then even with the 4 of the M scale, thus indicating that the difference between 10 and 6 is 4.

As already shown, the product of two numbers can be obtained (at least, approximately) by adding their logarithms and noting the number that has this sum of logarithms as its logarithm. Also, a quotient of two numbers can be found (approximately) by subtracting the logarithm of the divisor from the logarithm of the dividend and noting the number which has this difference of logarithms as its



Courtesy Keuffel & Esser Co.

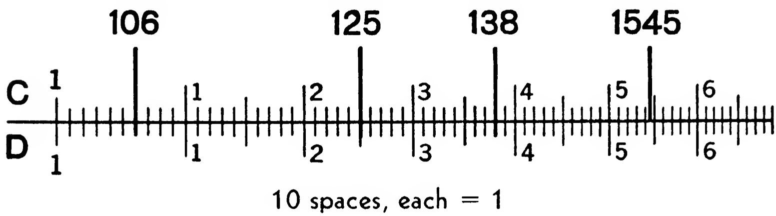
Fig. 86.

logarithm. The student is reminded that the sequence of significant figures in a product or quotient is determined by the mantissas of the logarithms involved. The slide rule is a device for performing mechanically the addition and subtraction of mantissas of logarithms involved in computing products and quotients. The manipulation of the slide rule is similar to the handling of the two 1-ft. rules described above.

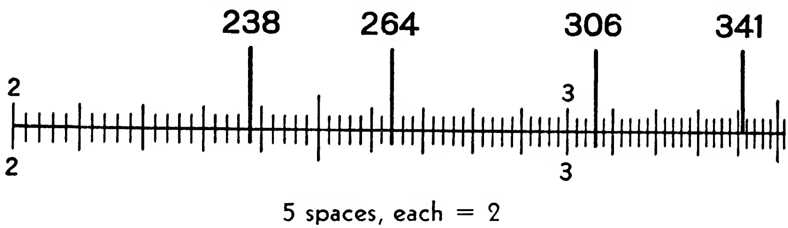
A slide rule consists essentially of two rules held alongside each other in such a manner that they are free to slide one upon the other in a lengthwise direction. There may be several graduated scales on the rules. Two of these

scales, the *C* scale on one rule and the *D* scale on the other, are exactly alike. Instead of being labeled "0," the left end of each of these scales is labeled "1," the number whose

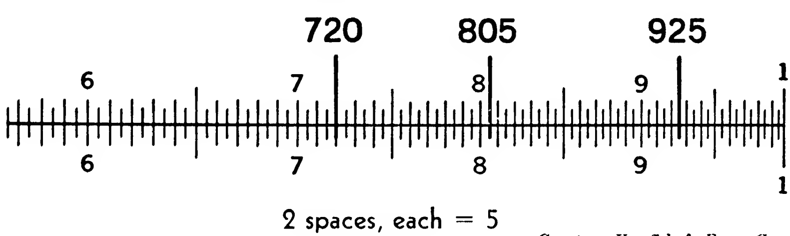
(a) The spaces between the secondary divisions between main divisions 1 and 2 are divided into tenths, so that each subdivision has a single value.



(b) The spaces between the secondary divisions between main divisions 2 and 3 and 3 and 4 are divided into fifths, so that each subdivision has a double value.



(c) The spaces between the secondary divisions for the remainder of the scale are divided only in half, so that each subdivision has a value of five.



Courtesy Keuffel & Esser Co.

Fig. 87.

logarithm is 0. The length of the scale is taken as unity, and the position of a number on the scale is determined by the mantissa of the logarithm of the number. The right end of the scale, also marked "1," may be regarded as cor-

responding to 1, 10, 100, or to any number whose logarithm has a mantissa equal to 0. Numbers having the same sequence of significant figures have a common mantissa in their logarithms; hence a position on the C or D scale may be associated with any number whose sequence of significant figures corresponds to that position. Since log 5 = 0.6990, the number 5 appears at a point about .7 of the way along the scale. Other numbers that may be associated with this point on the scale are .5, .05, 50, 500, and so forth.

Fig. 87, which illustrates sections of the C and D scales, shows how sequences of figures may be read on the scale.

Recalling the properties of logarithms and the manipulations with the two 1-ft. rules, the student will now see how the C and D scales can be used to add and subtract mantissas and to indicate thereby sequences of figures in required products and quotients. The position of the decimal point in a sequence of figures so obtained can be determined by inspection of the numbers involved in the computation. A more detailed explanation of the use of the slide rule will be given in connection with the examples that follow. The slide rule is usually equipped with a movable glass indicator across which there is a hairline perpendicular to the scales. This device is quite helpful to an operator in making settings and readings.

Multiplication

Examples

1. Find the product of 2 and 3.

Align the 1 of the left end of the C scale with the 2 of the D scale. The 3 of the C scale is then in line with the 6 of the D scale, indicating that the product of 2 and 3 is 6. In effect, the logarithms of 2 and 3 have been added, and the 6 is indicated as the number whose logarithm is the sum obtained.

2. Find the product of 4 and 3.

If the *left* 1 of the C scale is aligned with the 4 of the D scale, the 3 of the C scale falls beyond the right end of the D scale, thus

indicating that the sum of mantissas involved in this case is greater than 1. The point on the scale corresponding to the mantissa of the logarithm of the product can be found simply by shifting the C scale to the left its full length, thereby aligning the *right* 1 of the C scale with the 4 of the D scale. The 3 of the C scale is then in line with a point on the D scale that corresponds to 1.2, 12, 120, and so forth. Obviously, 12 is the proper reading in this case.

3. Multiply 23 by 7.

Align the *right* 1 of the C scale with the 7 of the D scale. The 23 of the C scale is then in line with the 161 of the D scale.

4. Multiply 2.3 by 7.

The alignment of the scales is the same as for multiplying 23 by 7. In this case, it is evident that the product is less than 100 and greater than 10; hence 16.1 is the proper reading of the product.

5. Multiply 38.4 by 75.

Align the *right* 1 of the C scale with 384 of the D scale. The 75 of the C scale is then in line with 288 of the D scale. The required product is evidently between 30×70 $(= 2100)$ and 40×80 $(= 3200)$; hence the correct reading of the product is 2880.

6. Multiply 3.84 by 7.5.

The alignment of the scales is the same as in the preceding example. In this case it is obvious that the result is near 4×7; hence 28.8 is the proper reading of the product.

Problems

By use of the slide rule obtain the following indicated products:

1. (a) (2)(4)
(b) (20)(4)
(c) (20)(40)
(d) (.2)(4)
(e) (.2)(.04)

2. (a) (6)(7)
(b) (600)(7)
(c) (6)(.7)
(d) (.06)(7)
(e) (.06)(.07)

3. (a) (23)(8)
(b) (2.3)(8)
(c) (230)(80)
(d) (.23)(.8)
(e) (.023)(80)

4. (a) (18)(35)
(b) (18)(54)
(c) (18)(65)

5. (a) (350)(23)
(b) (35)(752)
(c) (3.5)(75.2)

6. (3.25)(2.1)

7. $(\pi)(14)$

8. (525)(6.7)

9. 86% of 1935

10. (635)(.348)(1.5)

Division

Examples

1. Divide 6 by 3.

Align the 6 of the D scale with the 3 of the C scale. The *left* 1 of the C scale is then in line with the 2 of the D scale, indicating that the quotient is 2. In effect, the logarithm of 3 has been subtracted from the logarithm of 6, and the 2 is indicated as the number whose logarithm is the difference obtained.

2. Divide 35 by 5.

Align the 35 of the D scale with the 5 of the C scale. The *left* 1 of the C scale falls beyond the left end of the D scale, indicating that the mantissa of the logarithm of the divisor 5 is greater than the mantissa of the logarithm of the dividend 35. However, the position on the D scale corresponding to the mantissa of the logarithm of the quotient is now in line with the *right* 1 of the C scale. The quotient 7 is thus indicated.

3. Divide 2880 by 75.

Align the 288 of the D scale with the 75 of the C scale. The *right* 1 of the C scale is then in line with 384 of the D scale. Since $2800 \div 70 = 40$, the quotient $2880 \div 75$ is read as 38.4.

4. Divide .072 by 51.6.

Align 516 of the C scale with 72 of the D scale. The *left* 1 of the C scale is then in line with 1395 of the D scale. The quotient $.072 \div 51.6$ is near $.07 \div 50$ $(= .007 \div 5 = .0014)$. Hence .001395 is taken as the required quotient.

Problems

By use of the slide rule obtain the following indicated quotients:

1. (a) $8 \div 4$
(b) $9 \div 4.5$
(c) $80 \div 40$
(d) $.8 \div 4$
(e) $.8 \div .04$

2. (a) $75 \div 5$
(b) $7.5 \div 5$
(c) $750 \div 5$
(d) $.075 \div .5$
(e) $75 \div .05$

3. (a) $72 \div 144$
(b) $720 \div 144$
(c) $7.2 \div .144$

4. (a) $785 \div 54$
(b) $785 \div 5.4$
(c) $7.85 \div .054$

5. (a) $248 \div 62$
(b) $.248 \div .62$
(c) $.0248 \div 6.2$

6. $31.5 \div 16.2$

7. $3.14 \div 2.72$

8. $110 \div \pi$

9. $2875 \div 37.2$

10. $0.685 \div 8.93$

Multiplication and Division

Example

Solve the proportion $\frac{x}{6} = \frac{36}{24}$.

First obtain the result in the form

$$x = \frac{(36)(6)}{24}.$$

Then align 36 of the D scale with 24 of the C scale. The left 1 of the C scale is then in line with the quotient $36 \div 24$; hence the 6 of the C scale must be in line with $(6)(36 \div 24)$. The final result, 9, is thus read on the D scale from a single setting of the scales.

Problems

Determine the value of x *in each of the following, using the slide rule for multiplication and division:*

1. $\frac{x}{6} = \frac{42}{36}$.
2. $\frac{72}{56} = \frac{9}{x}$.
3. $\frac{x}{3.6} = \frac{2}{3}$.
4. 48 is $x\%$ of 64.
5. $35 = 62.5\%$ of x.
6. 16% of $x = 2\%$ of 192.
7. $\frac{x}{3.2} = \frac{4.6}{80}$.
8. $\frac{x}{24} = \frac{95}{25.2}$.
9. $x = \frac{(23)(7)}{(4)(12)}$.
10. $x = \frac{(4.1)(28)(92)}{(34)(.81)}$.

Squares and Square Roots

The construction of the A and B scales of the slide rule is similar to that of the C and D scales, except that the unit of length employed is half that used on the C and D scales. The A and D scales are on the *body* of the slide rule, while the B and C scales are on the *slide*. The numbers of the D scale are thus always aligned with their squares on the A scale. Similarly, the numbers of the C scale are in line with their squares on the B scale.

Examples

1. Find the square of 3.

Place the hairline of the indicator over 3 on the D scale and read 9, the square of 3, above on the A scale.

2. Find the square of 25.

Place the hairline over 25 of the D scale and read its square, 625, on the A scale.

3. Find the square of 56.4.

Place the hairline over 564 of the D scale and read 318 above on the A scale. Since $(50)^2 = 2500$ and $(60)^2 = 3600$, the square of 56.4 must have four places to the left of the decimal point. Then $(56.4)^2 = 3180$, approximately. Actually, $(56.4)^2 = 3180.96$.

4. Find the square root of 225.

Set the hairline over 225 of the *left* portion of the A scale and read 15 as the square root on the D scale.

5. Find the square root of 81.

Set the hairline over 81 of the *right* portion of the A scale and read 9 as the square root on the D scale.

In obtaining the square root of a number *greater than 1*, the *left* portion of the A scale is used if the number has an *odd* number of significant figures to the left of the decimal point; otherwise the *right* portion of the A scale is used.

In obtaining the square root of a *decimal fraction*, the *left* portion of the A scale is used if an *odd* number of zeros immediately follows the decimal point; otherwise the *right* portion of the A scale is used.

Problems

By use of the slide rule obtain the following indicated squares and square roots:

1. (a) $(4)^2$
(b) $(.4)^2$
(c) $(40)^2$

2. (a) $(18)^2$
(b) $(1.8)^2$
(c) $(.18)^2$

3. (a) $(24)^2$
(b) $(24.5)^2$
(c) $(2.45)^2$

4. (a) $(75)^2$
(b) $(7.5)^2$
(c) $(.75)^2$

5. (a) $(93)^2$
(b) $(930)^2$
(c) $(.093)^2$

6. (a) $\sqrt{9}$
(b) $\sqrt{.09}$
(c) $\sqrt{900}$

7. (a) $\sqrt{81}$
(b) $\sqrt{.81}$
(c) $\sqrt{8.1}$

8. (a) $\sqrt{49}$
(b) $\sqrt{.0049}$
(c) $\sqrt{490}$

9. (a) $\sqrt{256}$
(b) $\sqrt{2.56}$
(c) $\sqrt{25.6}$

10. (a) $\sqrt{73.6}$
(b) $\sqrt{642}$
(c) $\sqrt{.0386}$

11. (a) $\sqrt{837}$
(b) $\sqrt{6.25}$
(c) $\sqrt{.0324}$

12. (a) $\sqrt{1.69}$
(b) $\sqrt{57.4}$
(c) $\sqrt{72.45}$

TABLES

Tables

Table I. Measurements

LENGTH (LINEAR MEASURE)

English System

1 hand	=	4 inches (in.)
1 foot (ft.)	=	12 in.
1 vara	=	$33\frac{1}{3}$ in.
1 yard (yd.)	=	3 ft.
1 rod (rd.)	=	$5\frac{1}{2}$ yd.
	=	$16\frac{1}{2}$ ft.
1 chain (ch.)	=	4 rd.
	=	22 yd.
	=	66 ft.
1 mile (mi.)	=	80 ch
	=	320 rd.
	=	1760 yd.
	=	1900.8 varas
	=	5280 ft.

Metric System

1 centimeter (cm)	=	10 millimeters (mm)
1 decimeter (dm)	=	10 cm
1 meter (m)	=	10 dm
	=	100 cm
1 kilometer (km)	=	1000 m

Approximate Equivalents

1 in.	=	2.54 cm
1 yd.	=	.91 m
1 mi.	=	1.6 km
1 m	=	39.37 in.
	=	3.3 ft.

AREA (SQUARE MEASURE)

English System

1 square foot (sq. ft.)	=	144 square inches (sq. in.)
1 square foot (sq. ft.)	=	.000023 acre.
1 square yard (sq. yd.)	=	9 sq. ft.
1 square rod (sq. rd.)	=	$30\frac{1}{4}$ sq. yd.
1 square chain (sq. ch.)	=	16 sq. rd.
1 acre (A.)	=	10 sq. ch.
	=	160 sq. rd.
	=	4840 sq. yd.
	=	5645.376 sq. varas
	=	43,560 sq. ft.
	=	6,272,640 sq. in.
1 square mile (sq. mi.)	=	640 A.
1 section	=	640 A.
1 labor	=	1,000,000 sq. varas
	=	173 A.
1 square league	=	25 labors
	=	25,000,000 sq. varas
	=	4428.4 A.

Metric System

1 square centimeter (sq. cm or cm^2)	=	100 square millimeters (sq. mm or mm^2)
1 square decimeter (sq. dm or dm^2)	=	100 sq. cm
1 square meter (sq. m or m^2)	=	100 sq. dm

Table I. Measurements (Cont.)

VOLUME (CUBIC MEASURE)

English System

1 cubic foot (cu. ft.) = 1728 cubic inches (cu. in.)
1 cubic yard (cu. yd.) = 27 cu. ft.
1 cord (cd.) = 128 cu. ft.

Metric System

1 cubic centimeter (cc or cm^3) = 1000 cubic millimeters (cu. mm or mm^3)
1 cubic decimeter (cu. dm or dm^3) = 1000 cc
1 cubic meter (cu. m or m^3) = 1000 cu. dm
1 liter (l) = 1 cu. dm = 1000 cc

CAPACITY

Liquid Measure

1 fluid drachm (fʒ) = 60 minims (m.)
1 fluid ounce (f℥) = 8 fʒ
1 pint (pt.) = 16 f℥
= 4 gills (gi.)
1 quart (qt.) = 2 pt.
1 gallon (gal.) = 4 qt.
1 barrel (bbl.) = 31½ gal.
1 hogshead (hhd.) = 2 bbl.

Dry Measure

1 quart (qt.) = 2 pints (pt.)
1 small measure = 2 qt.
1 peck (pk.) = 8 qt.
1 bushel (bu.) = 4 pk.

Approximate Equivalents

1 bu. (stroked) contains 2150.4 cu. in.
1 bu. (stroked) contains 1¼ cu. ft.
1 cu. ft. = .8 bu.
1 cu. ft. contains 7½ gal.
1 dry quart contains 67⅕ cu. in.
1 liquid quart contains 57¾ cu. in.
1 gal. contains 231 cu. in.

1 teaspoonful = 4 cc
1 dessertspoonful = 8 cc
1 tablespoonful = 15 cc
1 cup = 120 cc
1 pt. = 2 cups
= 24 tablespoons

WEIGHT

Avoirdupois

1 ounce (oz.) = 437½ grains (gr.)
1 pound (lb.) = 16 oz.
1 hundredweight (cwt.) = 100 lb.
1 short ton (T.) = 2000 lb.
1 long ton = 2240 lb.

Apothecaries'

1 scruple (℈) = 20 grains (gr.)
1 drachm (ʒ) = 3 ℈
1 ounce (℥) = 8 ʒ
1 pound (lb.) = 12 ℥

Metric

1 gram (g) = weight of 1 cc of water at 4° C.
1 milligram (mg) = 0.001 g
1 centigram (cg) = 0.01 g
1 decigram (dg) = 0.1 g
1 kilogram (kg) = 1000 g
1 metric ton (t.) = 1,000,000 g

Approximate Equivalents

1 avoirdupois grain = 1 apothecaries' grain
1 g = .03527 oz.
1 g = 15.4324 gr.
1 oz. = 28.35 g
1 cu. ft. of water weighs 62½ lb.

Table I. Measurements (Cont.)

TIME

1 minute (min.) = 60 seconds (sec.)
1 hour (hr.) = 60 min.
1 day (da.) = 24 hr.
1 week (wk.) = 7 da.
1 month (mo.) = 30 da. (nominally, for ordinary accounting)
1 year (yr.) = 12 mo.
= 52 wk. (nearly)
= 365 da.
1 leap year = 366 da.
1 decade = 10 yr.
1 century = 100 yr.

TEMPERATURE

A change of 1° Centigrade (C.) is equivalent to a change of 1.8° Fahrenheit (F.).
A change of 1° F. is equivalent to a change of $\frac{5}{9}$ of 1° C.
Water freezes at 0° C., or 32° F.
Water boils at 100° C., or 212° F.

HEAT, ENERGY, WORK, AND POWER

1 small calorie = amount of heat required to raise 1 g of water 1° C.
1 large calorie = 1000 small calories
1 therm = 1000 large calories
1 foot-pound (ft.-lb.) = work done by force of 1 lb. acting through a distance of 1 ft.
Power = rate of doing work
1 horsepower (h.p.) = 550 ft.-lb. per second
= 746 watts (w)
1 kilowatt (kw) = 1000 w
1 kilowatt-hour (kw-hr.) = energy produced by 1 kw of power acting for 1 hr.

COUNTING

1 dozen (doz.) = 12 units
1 gross (gro.) = 12 doz.
1 great gross = 12 gro.
1 score = 20 units

ANGLES

1 minute (′) = 60 seconds (″)
1 degree (°) = 60′
1 radian = 57.3° (nearly)
1 right angle (∟) = 90°
= $\pi/2$ radians
1 straight angle (st. ∠) = 180°
= π radians
1 revolution (rev.) = 360°
= 2π radians
π = 3.1416 or $3\frac{1}{7}$, approx.

Table II. Convenient Equivalents*

CONSTRUCTION MATERIALS

Brick

1 brick is considered as measuring 2 in. × 4 in. × 8 in.
1 cu. ft. brick work contains 22 bricks.

Concrete

1 bag cement occupies .94 cu. ft.
1 bag cement weighs 94 lb.
1 bbl. cement contains 4 bags.
1 bbl. cement occupies 3.76 cu. ft.
1 bbl. cement weighs 376 lb.
1 cu. ft. cement weighs 100 lb.
1 cu. ft. concrete weighs 140 lb.
1 cu. ft. crushed stone weighs 100 lb.
1 cu. ft. sand weighs 105 lb.

Iron

1 cu. ft. cast iron weighs 450 lb.
1 cu. ft. wrought iron weighs 480 lb.
1 cu. ft. steel weighs 490 lb.

Lumber

1 cu. ft. red oak weighs 45 lb.
1 cu. ft. white oak weighs 46 lb.
1 cu. ft. white pine weighs 27 lb.
1 cu. ft. yellow pine weighs 41 lb.
1 board foot (bd. ft.) is the equivalent of a piece of lumber 1 ft. square and 1 in. (or less) thick.
1 sq. ft. of roof requires ten 4-in. shingles
1 square of roofing equals 100 sq. ft.

Paint

1 gal. paint covers 300 sq. ft., two coats.

PRODUCE

1 bu. grain occupies $1\frac{1}{4}$ cu. ft.
1 bu. ear corn, without husk, occupies $2\frac{1}{2}$ cu. ft.
1 bu. ear corn, with husk, occupies $3\frac{1}{2}$ cu. ft.
1 T. hay occupies 500 cu. ft.
1 T. silage occupies 50 cu. ft.
1 gal. 20% cream weighs 8.4 lb.
1 gal. 22% cream weighs 8.339 lb.
1 gal. skim milk weighs 8.65 lb.
1 gal. 3% milk weighs 8.60 lb.
1 gal. average milk weighs 8.6 lb.
1 gal. 4% milk weighs 8.59 lb.
1 gal. 5% milk weighs 8.58 lb.
1 gal. water weighs 8.339 lb.

* Many of these equivalents are approximations.

Table III. Weights of Commodities per Bushel*

Commodity	Weight (lb.)	Commodity	Weight (lb.)
Alfalfa seed	60	Milo maize	50
Apples	48	Oats	32
Barley	48	Onions	57
Beans, snap	30	Peaches	50
Beans, dry	60	Peanuts, roasted	20
Beans, castor	46	Peanuts, Spanish	24
Bran	20	Pears	58
Buckwheat	52	Peas, dried	60
Clover seed	60	Peas, green, in pod	32
Coal, anthracite	80	Plums	56
Broom-corn seed	48	Prunes	56
Cherries, with stems	56	Popcorn, ear	70
Clover seed	60	Popcorn, shelled	56
Corn meal, unbolted	50	Potatoes, Irish	60
Corn, ear, new crop before Dec. 1	72	Potatoes, sweet	50
		Rye	56
Corn, ear, after Dec. 1	70	Salt, coarse	55
Corn, shelled	56	Salt, fine	50
Cottonseed	32	Shorts	20
Cowpeas	60	Sorghum seed	50
Cranberries	33	Spinach	18
Cucumbers	48	Sudan grass seed	40
Flaxseed	56	Timothy seed	45
Grapes	48	Tomatoes	56
Kafir	50	Turnips	55
Millet	50	Wheat	60

* There is some variation in legal weights of certain commodities per bushel in different states.

Table IV. Proportions of Cement, Sand, and Gravel in Concrete Mixtures

Description of Mixture	Use	Proportion
Rich	Structural parts under heavy stress, and watertight structures	1–1½–3
Standard	Reinforced concrete under considerable stress (floors, beams, and columns)	1–2–4
Medium	Plain concrete of moderate strength (walls and sidewalks)	1–2½–5
Lean	Concrete under compression only	1–3–6

Table V. Approximate Number of Board Feet of Lumber in Trees Containing One, Two, or Three 16-ft. Merchantable Logs*

Diameter of Tree at 4½ ft. Above Ground (in.)	Board Feet in Tree Having One 16-ft. Log	Board Feet in Tree Having Two 16-ft. Logs	Board Feet in Tree Having Three 16-ft. Logs
8	19	32	—
9	27	43	—
10	36	57	64
11	46	73	84
12	58	90	108
13	68	110	131
14	80	130	160
15	92	154	192
16	106	181	226
17	120	208	260
18	136	234	305
19	150	263	350
20	169	300	396
21	180	335	440
22	204	365	485
23	225	395	530
24	246	430	580

* This table was made primarily for use on Southern pine trees, but it may be used in estimating the board-foot content of straight trunks and limbs of other kinds of trees.

Table VI. Per Cent of Nitrogen, Phosphoric Acid, and Potash in Common Fertilizer Materials

Type of Material	Fertilizing Material	Nitrogen (%)	Available Phosphoric Acid (%)	Potash (%)
Nitrogenous	Cyanamid	21	0	0
	Dried blood	13	0	0
	Sodium nitrate	16	0	0
	Ammonium sulphate	20	0	0
Phosphatic	Acid phosphate	0	16	0
	Basic slag	0	15	0
	Phosphate of lime	0	12	0
	Superphosphate (18%)	0	18	0
	Superphosphate (20%)	0	20	0
	Superphosphate (32%)	0	32	0
	Superphosphate (45%)	0	45	0
Potash	Muriate of potash	0	0	50
	Kainit	0	0	12
	Sulphate of potash	0	0	48
Combined nitrogenous and phosphatic	Bone meal	4	23	0
	Ground tankage	8	10	0
	Fish scrap	8	7	0
Combined phosphatic and potash	Wood ashes	0	1	6
Combined nitrogenous, phosphatic, and potash	Cottonseed	3.13	1.27	1.17
	Cottonseed meal	7	2.5	1.5
	Farmyard manure	.5	.5	.5
	Hen manure (dry)	2	2	1
	Pea straw	.1	.3	1
	Leaves	.7	.15	.3

Table VII. Average Percentage Composition of Feeds

Feeds	Protein	Ether Extract	Crude Fiber	Nitrogen-Free Extract	Water	Ash	Digestible Protein (%)
Alfalfa hay	14.8	2.0	29.1	37.4	8.3	8.4	11.0
Alfalfa leaf meal	21.5	2.6	15.8	40.5	7.7	11.9	16.1
Alfalfa meal	14.6	1.8	29.9	36.8	8.6	8.3	10.9
Barley chops (grain)	12.0	2.1	6.3	67.5	9.3	2.8	9.6
Beet pulp, dried	9.0	.6	19.3	59.0	8.5	3.6	4.5
Bermuda hay	5.9	1.5	26.7	50.3	7.8	7.8	3.0
Bone meal, raw	25.5	2.2	1.2	4.2	5.2	61.7	21.2
Buttermilk, dried	34.4	5.0	.4	40.6	8.2	11.4	32.6
Corn, grain	10.4	4.4	2.3	72.5	9.1	1.3	6.4
Corn cob	3.1	.5	33.0	54.0	7.3	2.1	.3
Corn feed meal	9.9	4.4	3.0	70.3	10.7	1.7	6.4
Corn meal	10.1	3.5	1.7	73.1	10.4	1.2	6.5
Cottonseed	20.9	17.9	23.8	26.9	7.0	3.5	15.4
Cottonseed cake	43.0	6.0	12.0	23.0	—	—	34.5
Cottonseed meal	43.0	6.0	12.0	23.0	—	—	35.7
Cottonseed hulls	4.1	.9	47.6	35.3	9.4	2.7	.4
Cowpea hay	13.1	2.9	30.6	33.9	9.6	9.9	9.0
Feterita heads	10.6	2.7	7.4	65.9	10.2	3.2	8.1
Hegari, grain or chops	11.4	2.1	2.4	70.8	11.8	1.5	8.8
Johnson grass hay	6.1	1.7	29.1	45.6	7.4	10.1	2.7
Kafir grain or chops	11.2	2.9	2.3	71.1	10.5	2.0	8.6
Linseed meal	32.2	10.2	9.9	37.6	5.3	4.8	27.0
Meat scraps	61.2	11.8	3.2	1.7	6.2	15.9	60.5
Milk, dried, skimmed	35.6	.3	.1	51.3	5.1	7.6	33.5
Milo heads	10.1	2.5	6.8	67.1	10.0	3.5	7.6
Molasses, blackstrap	2.4	.0	.0	65.0	—	—	.3
Oats, whole ground	12.2	4.6	11.4	58.9	9.1	3.8	9.6
Oatmeal	14.9	5.4	3.5	65.8	8.2	2.2	13.4
Peanut hay	10.0	3.5	24.0	44.0	—	—	—
Rice, ground whole	7.6	1.9	9.2	64.8	11.5	5.0	5.8
Rice bran	12.8	13.1	12.7	41.7	9.0	10.7	8.9
Sorghum hay or fodder	5.7	2.2	19.0	48.5	18.6	6.0	1.8
Sorghum and corn silage	4.9	1.6	13.0	36.8	39.6	4.1	.8
Sudan grass hay	8.6	1.8	29.7	43.4	8.4	8.1	4.5
Tankage	48.6	9.7	2.8	2.9	6.7	29.3	33.8
Wheat	14.0	1.7	3.0	69.4	10.0	1.9	11.3
Wheat bran	16.9	4.0	8.8	54.9	9.7	5.7	13.3
Wheat gray shorts	18.0	4.5	5.6	57.8	10.0	4.1	14.9

Table VIII. Number of Pounds of Digestible Nutrients in 100 lb. of Feed*

Feed	Protein	Ether Extract	Crude Fiber	Nitrogen-Free Extract	Total**	Productive Energy per 100 lb. (therms)
Alfalfa hay	11.0	0.7	13.0	26.5	52.1	37.4
Alfalfa leaf meal	16.1	0.8	8.7	32.4	59.0	52.3
Alfalfa meal	10.9	0.5	14.0	26.0	52.0	41.2
Barley (grain)	9.6	1.8	4.4	62.7	80.7	74.4
Beet pulp, dried	4.5	0	16.2	49.0	69.7	75.1
Bermuda hay	3.0	0.7	14.3	26.0	44.9	31.3
Corn, grain	6.4	3.8	0.7	66.7	82.3	84.8
Corn cob	0.3	0.1	18.1	26.4	45.0	13.6
Corn meal	6.5	3.3	1.0	68.6	83.5	86.8
Cottonseed	15.4	15.6	18.0	13.2	81.7	76.6
Cottonseed meal	35.9	7.2	3.9	17.8	73.8	74.9
Cottonseed hulls	0.4	0.7	23.2	18.5	43.7	17.9
Cowpea hay	9.0	1.2	14.3	23.1	49.1	32.6
Kafir grain or chops	8.6	2.2	0.6	63.1	77.2	85.8
Linseed meal	27.0	9.4	5.9	30.8	84.8	85.0
Milk, dried, skimmed	33.5	0.3	0.1	50.3	84.6	85.8
Milo heads	7.6	2.2	3.5	60.9	76.9	77.5
Molasses, blackstrap	0.3	0	0	57.6	57.9	62.8
Oats, whole ground	9.6	3.9	4.7	48.3	81.4	71.9
Peanut hay	6.5	2.1	12.2	33.3	56.7	45.2
Rice, ground whole	5.8	1.4	1.0	58.8	69.7	70.2
Rice bran	8.9	10.8	2.9	31.5	67.6	69.9
Sorghum hay or fodder	1.8	1.3	11.5	28.6	44.8	35.7
Sorghum and corn silage	0.8	0.9	6.4	24.2	33.4	28.6
Sudan grass hay	4.5	0.8	19.3	23.7	49.3	33.9
Tankage	33.8	8.8	2.0	2.2	57.8	59.7
Wheat	11.3	1.3	1.8	65.2	81.2	78.8
Wheat bran	13.3	2.7	2.8	39.5	61.7	56.8
Wheat gray shorts	14.9	4.0	2.0	52.2	78.1	75.7

* Computed from data given in Bulletins 329, 402, 454, and 461 of the Texas Agricultural Experiment Station

** Totals include 2.25 times digestible ether extracts.

Table IX. Tentative Standards for Feeding per Day per 1000 lb. Live Weight*

Animal	Per Day per 1000 lb. Live Weight		
	Dry Matter (lb.)	Digestible Crude Protein (lb.)	Productive Value (therms)
Growing dairy cattle:			
Weight 100-200 lb.	22.0-24.0	2.8 -3.1	15.6 -17.5
Weight 200-300 lb.	23.0-25.0	2.5 -2.8	15.1 -17.0
Weight 300-400 lb.	24.0-26.0	2.2 -2.5	14.2 -16.0
Weight 400-500 lb.	22.0-25.0	1.9 -2.2	13.3 -15.0
Weight 500-600 lb.	21.5-24.5	1.7 -1.9	12.6 -14.5
Weight 600-700 lb.	21.0-24.0	1.6 -1.8	12.0 -13.8
Weight 700-800 lb.	20.5-23.5	1.5 -1.7	11.0 -13.0
Weight 800-900 lb.	20.0-23.0	1.4 -1.6	10.4 -12.3
Weight 900-1000 lb.	20.0-23.0	1.2 -1.9	9.7 -11.4
Growing steers with some fattening:			
Weight 100-200 lb.	22.0-24.0	2.8 -3.1	15.8 -17.7
Weight 200-300 lb.	23.0-25.0	2.5 -2.8	15.6 -17.5
Weight 300-400 lb.	24.0-26.0	2.2 -2.5	14.5 -16.4
Weight 400-500 lb.	24.0-26.0	2.0 -2.2	14.0 -15.9
Weight 500-600 lb.	23.0-25.0	1.9 -2.1	13.7 -15.5
Weight 600-700 lb.	22.0-24.0	1.8 -2.0	13.3 -15.2
Weight 700-800 lb.	21.0-23.0	1.7 -1.9	13.0 -14.9
Weight 800-900 lb.	20.5-22.5	1.6 -1.8	12.6 -14.5
Weight 900-1000 lb.	20.0-22.0	1.5 -1.7	12.3 -14.1
Weight 1000-1100 lb.	19.5-21.5	1.4 -1.6	11.8 -13.7
Weight 1100-1200 lb.	19.0-21.0	1.3 -1.5	11.4 -13.3
Fattening 2-yr.-old steers on full feed:			
First 40-60 da.	22.0-28.0	1.7 -2.0	14.3 -16.2
Second 40-60 da.	20.0-30.0	1.6 -1.9	13.2 -15.7
Third 40-60 da.	18.0-28.0	1.5 -1.8	13.0 -15.3
Ox at rest in stall	13.0-21.0	0.5 -0.7	6.8 - 7.8
Wintering beef cows and calves	14.0-25.0	0.7 -0.8	8.0 -10.0
Horses:			
Idle	13.0-19.0	0.8 -1.0	6.5 - 8.4
At light work	15.0-21.0	1.0 -1.2	8.4 -10.5
At medium work	16.0-22.0	1.2 -1.5	10.0 -13.0
At heavy work	18.0-24.0	1.5 -2.0	13.0 -16.0

* Standards taken from Bulletins 454 and 461 of the Texas Agricultural Experiment Station.

Table IX. Tentative Standards for Feeding per Day per 1000 lb. Live Weight (Cont.)

Animal	Per Day per 1000 lb. Live Weight: Dry Matter (lb.)	Digestible Crude Protein (lb.)	Productive Value (therms)
Brood mares suckling foals, but not at work	15.0-22.0	1.2 -1.4	8.4 -11.2
Growing colts, over 6 mo.	18.0-22.0	1.6 -1.8	10.0 -12.0
Fattening lambs:			
Weight 50-70 lb.	27.0-30.0	2.6 -3.0	18.0 -20.5
Weight 70-90 lb.	28.0-31.0	2.4 -2.7	18.0 -21.4
Weight 90-100 lb.	27.0-31.0	2.2 -2.4	18.0 -21.4
Fattening sheep	24.0-32.0	1.6 -2.0	15.0 -16.0
Dairy cows:			
For maintenance of 1000-lb. cow		0.60	6.0
To allowance for maintenance add:			
For each pound of 3.0% milk		0.045- .055	0.25
For each pound of 4.0% milk		0.053- .065	0.30
For each pound of 5.0% milk		0.060- .070	0.35
For each pound of 6.0% milk		0.065- .080	0.40
For each pound of 7.0% milk		0.070- .085	0.45
Sheep maintaining, mature:			
Coarse wool	18.0-23.0	1.0 -1.3	9.5 -12.0
Fine wool	20.0-26.0	1.1 -1.4	10.5 -13.0
Breeding ewes, with lambs	23.0-27.0	2.5 -2.8	16.7 -18.6
Fattening hogs:			
Weight 30-50 lb.	44.0-63.0	7.0 -8.0	35.0 -50.0
Weight 50-100 lb.	33.0-43.0	5.3 -6.0	30.0 -34.0
Weight 100-150 lb.	30.0-41.0	4.4 -5.0	26.0 -33.0
Weight 150-200 lb.	28.0-38.0	3.4 -4.2	24.0 -31.0
Weight 200-250 lb.	25.0-36.0	2.9 -3.8	21.0 -29.0
Weight 250-300 lb.	20.0-32.0	2.6 -3.4	18.0 -26.0
Brood sows with pigs	20.0-28.0	2.4 -3.0	16.0 -24.0

Table X. Trigonometric Functions*

Angle	Sine	Tangent	Cotangent	Cosine	
0° 00′	.0000	.0000	——	1.0000	**90° 00′**
10	.0029	.0029	343.77	1.0000	50
20	.0058	.0058	171.89	1.0000	40
30	.0087	.0087	114.59	1.0000	**30**
40	.0116	.0116	85.940	.9999	20
50	.0145	.0145	68.750	.9999	10
1° 00′	.0175	.0175	57.290	.9998	**89° 00′**
10	.0204	.0204	49.104	.9998	50
20	.0233	.0233	42.964	.9997	40
30	.0262	.0262	38.188	.9997	**30**
40	.0291	.0291	34.368	.9996	20
50	.0320	.0320	31.242	.9995	10
2° 00′	.0349	.0349	28.636	.9994	**88° 00′**
10	.0378	.0378	26.432	.9993	50
20	.0407	.0407	24.542	.9992	40
30	.0436	.0437	22.904	.9990	**30**
40	.0465	.0466	21.470	.9989	20
50	.0494	.0495	20.206	.9988	10
3° 00′	.0523	.0524	19.081	.9986	**87° 00′**
10	.0552	.0553	18.075	.9985	50
20	.0581	.0582	17.169	.9983	40
30	.0610	.0612	16.350	.9981	**30**
40	.0640	.0641	15.605	.9980	20
50	.0669	.0670	14.924	.9978	10
4° 00′	.0698	.0699	14.301	.9976	**86° 00′**
10	.0727	.0729	13.727	.9974	50
20	.0756	.0758	13.197	.9971	40
30	.0785	.0787	12.706	.9969	**30**
40	.0814	.0816	12.251	.9967	20
50	.0843	.0846	11.826	.9964	10
5° 00′	.0872	.0875	11.430	.9962	**85° 00′**
	Cosine	Cotangent	Tangent	Sine	Angle

* Adapted from W W Elliott and E. Roy C. Miles, *College Mathematics: A First Course.* New York: Prentice-Hall, Inc , 1940.

Table X. Trigonometric Functions (Cont.)

Angle	Sine	Tangent	Cotangent	Cosine	
5° 00′	.0872	.0875	11.430	.9962	**85° 00′**
10	.0901	.0904	11.059	.9959	50
20	.0929	.0934	10.712	.9957	40
30	.0958	.0963	10.385	.9954	**30**
40	.0987	.0992	10.078	.9951	20
50	.1016	.1022	9.7882	.9948	10
6° 00′	.1045	.1051	9.5144	.9945	**84° 00′**
10	.1074	.1080	9.2553	.9942	50
20	.1103	.1110	9.0098	.9939	40
30	.1132	.1139	8.7769	.9936	**30**
40	.1161	.1169	8.5555	.9932	20
50	.1190	.1198	8.3450	.9929	10
7° 00′	.1219	.1228	8.1443	.9925	**83° 00′**
10	.1248	.1257	7.9530	.9922	50
20	.1276	.1287	7.7704	.9918	40
30	.1305	.1317	7.5958	.9914	**30**
40	.1334	.1346	7.4287	.9911	20
50	.1363	.1376	7.2687	.9907	10
8° 00′	.1392	.1405	7.1154	.9903	**82° 00′**
10	.1421	.1435	6.9682	.9899	50
20	.1449	.1465	6.8269	.9894	40
30	.1478	.1495	6.6912	.9890	**30**
40	.1507	.1524	6.5606	.9886	20
50	.1536	.1554	6.4348	.9881	10
9° 00′	.1564	.1584	6.3138	.9877	**81° 00′**
10	.1593	.1614	6.1970	.9872	50
20	.1622	.1644	6.0844	.9868	40
30	.1650	.1673	5.9758	.9863	**30**
40	.1679	.1703	5.8708	.9858	20
50	.1708	.1733	5.7694	.9853	10
10° 00′	.1736	.1763	5.6713	.9848	**80° 00′**
	Cosine	Cotangent	Tangent	Sine	Angle

Table X. Trigonometric Functions (Cont.)

Angle	Sine	Tangent	Cotangent	Cosine	
10° 00′	.1736	.1763	5.6713	.9848	**80° 00′**
10	.1765	.1793	5.5764	.9843	50
20	.1794	.1823	5.4845	.9838	40
30	.1822	.1853	5.3955	.9833	**30**
40	.1851	.1883	5.3093	.9827	20
50	.1880	.1914	5.2257	.9822	10
11° 00′	.1908	.1944	5.1446	.9816	**79° 00′**
10	.1937	.1974	5.0658	.9811	50
20	.1965	.2004	4.9894	.9805	40
30	.1994	.2035	4.9152	.9799	**30**
40	.2022	.2065	4.8430	.9793	20
50	.2051	.2095	4.7729	.9787	10
12° 00′	.2079	.2126	4.7046	.9781	**78° 00′**
10	.2108	.2156	4.6382	.9775	50
20	.2136	.2186	4.5736	.9769	40
30	.2164	.2217	4.5107	.9763	**30**
40	.2193	.2247	4.4494	.9757	20
50	.2221	.2278	4.3897	.9750	10
13° 00′	.2250	.2309	4.3315	.9744	**77° 00′**
10	.2278	.2339	4.2747	.9737	50
20	.2306	.2370	4.2193	.9730	40
30	.2334	.2401	4.1653	.9724	**30**
40	.2363	.2432	4.1126	.9717	20
50	.2391	.2462	4.0611	.9710	10
14° 00′	.2419	.2493	4.0108	.9703	**76° 00′**
10	.2447	.2524	3.9617	.9696	50
20	.2476	.2555	3.9136	.9689	40
30	.2504	.2586	3.8667	.9681	**30**
40	.2532	.2617	3.8208	.9674	20
50	.2560	.2648	3.7760	.9667	10
15° 00′	.2588	.2679	3.7321	.9659	**75° 00′**
	Cosine	Cotangent	Tangent	Sine	Angle

Table X. Trigonometric Functions (Cont.)

Angle	Sine	Tangent	Cotangent	Cosine	
15° 00′	.2588	.2679	3.7321	.9659	**75° 00′**
10	.2616	.2711	3.6891	.9652	50
20	.2644	.2742	3.6470	.9644	40
30	.2672	.2773	3.6059	.9636	**30**
40	.2700	.2805	3.5656	.9628	20
50	.2728	.2836	3.5261	.9621	10
16° 00′	.2756	.2867	3.4874	.9613	**74° 00′**
10	.2784	.2899	3.4495	.9605	50
20	.2812	.2931	3.4124	.9596	40
30	.2840	.2962	3.3759	.9588	**30**
40	.2868	.2994	3.3402	.9580	20
50	.2896	.3026	3.3052	.9572	10
17° 00′	.2924	.3057	3.2709	.9563	**73° 00′**
10	.2952	.3089	3.2371	.9555	50
20	.2979	.3121	3.2041	.9546	40
30	.3007	.3153	3.1716	.9537	**30**
40	.3035	.3185	3.1397	.9528	20
50	.3062	.3217	3.1084	.9520	10
18° 00′	.3090	.3249	3.0777	.9511	**72° 00′**
10	.3118	.3281	3.0475	.9502	50
20	.3145	.3314	3.0178	.9492	40
30	.3173	.3346	2.9887	.9483	**30**
40	.3201	.3378	2.9600	.9474	20
50	.3228	.3411	2.9319	.9465	10
19° 00′	.3256	.3443	2.9042	.9455	**71° 00′**
10	.3283	.3476	2.8770	.9446	50
20	.3311	.3508	2.8502	.9436	40
30	.3338	.3541	2.8239	.9426	**30**
40	.3365	.3574	2.7980	.9417	20
50	.3393	.3607	2.7725	.9407	10
20° 00′	.3420	.3640	2.7475	.9397	**70° 00′**
	Cosine	Cotangent	Tangent	Sine	Angle

Table X. Trigonometric Functions (Cont.)

Angle	Sine	Tangent	Cotangent	Cosine	
20° 00′	.3420	.3640	2.7475	.9397	**70° 00′**
10	.3448	.3673	2.7228	.9387	50
20	.3475	.3706	2.6985	.9377	40
30	.3502	.3739	2.6746	.9367	**30**
40	.3529	.3772	2.6511	.9356	20
50	.3557	.3805	2.6279	.9346	10
21° 00′	.3584	.3839	2.6051	.9336	**69° 00′**
10	.3611	.3872	2.5826	.9325	50
20	.3638	.3906	2.5605	.9315	40
30	.3665	.3939	2.5386	.9304	**30**
40	.3692	.3973	2.5172	.9293	20
50	.3719	.4006	2.4960	.9283	10
22° 00′	.3746	.4040	2.4751	.9272	**68° 00′**
10	.3773	.4074	2.4545	.9261	50
20	.3800	.4108	2.4342	.9250	40
30	.3827	.4142	2.4142	.9239	**30**
40	.3854	.4176	2.3945	.9228	20
50	.3881	.4210	2.3750	.9216	10
23° 00′	.2907	.4245	2.3559	.9205	**67° 00′**
10	.3934	.4279	2.3369	.9194	50
20	.3961	.4314	2.3183	.9182	04
30	.3987	.4348	2.2998	.9171	**30**
40	.4014	.4383	2.2817	.9159	20
50	.4041	.4417	2.2637	.9147	10
24° 00′	.4067	.4452	2.2460	.9135	**66° 00′**
10	.4094	.4487	2.2286	.9124	50
20	.4120	.4522	2.2113	.9112	40
30	.4147	.4557	2.1943	.9100	**30**
40	.4173	.4592	2.1775	.9088	20
50	.4200	.4628	2.1609	.9075	10
25° 00′	.4226	.4663	2.1445	.9063	**65° 00′**
	Cosine	Cotangent	Tangent	Sine	Angle

Table X. Trigonometric Functions (Cont.)

Angle	Sine	Tangent	Cotangent	Cosine	
25° 00′	.4226	.4663	2.1445	.9063	**65° 00′**
10	.4253	.4699	2.1283	.9051	50
20	.4279	.4734	2.1123	.9038	40
30	.4305	.4770	2.0965	.9026	**30**
40	.4331	.4806	2.0809	.9013	20
50	.4358	.4841	2.0655	.9001	10
26° 00′	.4384	.4877	2.0503	.8988	**64° 00′**
10	.4410	.4913	2.0353	.8975	50
20	.4436	.4950	2.0204	.8962	40
30	.4462	.4986	2.0057	.8949	**30**
40	.4488	.5022	1.9912	.8936	20
50	.4514	.5059	1.9768	.8923	10
27° 00′	.4540	.5095	1.9626	.8910	**63° 00′**
10	.4566	.5132	1.9486	.8897	50
20	.4592	.5169	1.9347	.8884	40
30	.4617	.5206	1.9210	.8870	**30**
40	.4643	.5243	1.9074	.8857	20
50	.4669	.5280	1.8940	.8843	10
28° 00′	.4695	.5317	1.8807	.8829	**62° 00′**
10	.4720	.5354	1.8676	.8816	50
20	.4746	.5392	1.8546	.8802	40
30	.4772	.5430	1.8418	.8788	**30**
40	.4797	.5467	1.8291	.8774	20
50	.4823	.5505	1.8165	.8760	10
29° 00′	.4848	.5543	1.8040	.8746	**61° 00′**
10	.4874	.5581	1.7917	.8732	50
20	.4899	.5619	1.7796	.8718	40
30	.4924	.5658	1.7675	.8704	**30**
40	.4950	.5696	1.7556	.8689	20
50	.4975	.5735	1.7437	.8675	10
30° 00′	.5000	.5774	1.7321	.8660	**60° 00′**
	Cosine	Cotangent	Tangent	Sine	Angle

Table X. Trigonometric Functions (Cont.)

Angle	Sine	Tangent	Cotangent	Cosine	
30° 00′	.5000	.5774	1.7321	.8660	**60° 00′**
10	.5025	.5812	1.7205	.8646	50
20	.5050	.5851	1.7090	.8631	40
30	.5075	.5890	1.6977	.8616	**30**
40	.5100	.5930	1.6864	.8601	20
50	.5125	.5969	1.6753	.8587	10
31° 00′	.5150	.6009	1.6643	.8572	**59° 00′**
10	.5175	.6048	1.6534	.8557	50
20	.5200	.6088	1.6426	.8542	40
30	.5225	.6128	1.6319	.8526	**30**
40	.5250	.6168	1.6212	.8511	20
50	.5275	.6208	1.6107	.8496	10
32° 00′	.5299	.6249	1.6003	.8480	**58° 00′**
10	.5324	.6289	1.5900	.8465	50
20	.5348	.6330	1.5798	.8450	40
30	.5373	.6371	1.5697	.8434	**30**
40	.5398	.6412	1.5597	.8418	20
50	.5422	.6453	1.5497	.8403	10
33° 00′	.5446	.6494	1.5399	.8387	**57° 00′**
10	.5471	.6536	1.5301	.8371	50
20	.5495	.6577	1.5204	.8355	40
30	.5519	.6619	1.5108	.8339	**30**
40	.5544	.6661	1.5013	.8323	20
50	.5568	.6703	1.4919	.8307	10
34° 00′	.5592	.6745	1.4826	.8290	**56° 00′**
10	.5616	.6787	1.4733	.8274	50
20	.5640	.6830	1.4641	.8258	40
30	.5664	.6873	1.4550	.8241	**30**
40	.5688	.6916	1.4460	.8225	20
50	.5712	.6959	1.4370	.8208	10
35° 00′	.5736	.7002	1.4281	.8192	**55° 00′**
	Cosine	Cotangent	Tangent	Sine	Angle

Table X. Trigonometric Functions (Cont.)

Angle	Sine	Tangent	Cotangent	Cosine	
35° 00′	.5736	.7002	1.4281	.8192	**55° 00′**
10	.5760	.7046	1.4193	.8175	50
20	.5783	.7089	1.4106	.8158	40
30	.5807	.7133	1.4019	.8141	**30**
40	.5831	.7177	1.3934	.8124	20
50	.5854	.7221	1.3848	.8107	10
36° 00′	.5878	.7265	1.3764	.8090	**54° 00′**
10	.5901	.7310	1.3680	.8073	50
20	.5925	.7355	1.3597	.8056	40
30	.5948	.7400	1.3514	.8039	**30**
40	.5972	.7445	1.3432	.8021	20
50	.5995	.7490	1.3351	.8004	10
37° 00′	.6018	.7536	1.3270	.7986	**53° 00′**
10	.6041	.7581	1.3190	.7969	50
20	.6065	.7627	1.3111	.7951	40
30	.6088	.7673	1.3032	.7934	**30**
40	.6111	.7720	1.2954	.7916	20
50	.6134	.7766	1.2876	.7898	10
38° 00′	.6157	.7813	1.2799	.7880	**52° 00′**
10	.6180	.7860	1.2723	.7862	50
20	.6202	.7907	1.2647	.7844	40
30	.6225	.7954	1.2572	.7826	**30**
40	.6248	.8002	1.2497	.7808	20
50	.6271	.8050	1.2423	.7790	10
39° 00′	.6293	.8098	1.2349	.7771	**51° 00′**
10	.6316	.8146	1.2276	.7753	50
20	.6338	.8195	1.2203	.7735	40
30	.6361	.8243	1.2131	.7716	**30**
40	.6383	.8292	1.2059	.7698	20
50	.6406	.8342	1.1988	.7679	10
40° 00′	.6428	.8391	1.1918	.7660	**50° 00′**
	Cosine	Cotangent	Tangent	Sine	Angle

Table X. Trigonometric Functions (Cont.)

Angle	Sine	Tan-gent	Cotan-gent	Cosine	
40° 00′	.6428	.8391	1.1918	.7660	**50° 00′**
10	.6450	.8441	1.1847	.7642	50
20	.6472	.8491	1.1778	.7623	40
30	.6494	.8541	1.1708	.7604	**30**
40	.6517	.8591	1.1640	.7585	20
50	.6539	.8642	1.1571	.7566	10
41° 00′	.6561	.8693	1.1504	.7547	**49° 00′**
10	.6583	.8744	1.1436	.7528	50
20	.6604	.8796	1.1369	.7509	40
30	.6626	.8847	1.1303	.7490	**30**
40	.6648	.8899	1.1237	.7470	20
50	.6670	.8952	1.1171	.7451	10
42° 00′	.6691	.9004	1.1106	.7431	**48° 00′**
10	.6713	.9057	1.1041	.7412	50
20	.6734	.9110	1.0977	.7392	40
30	.6756	.9163	1.0913	.7373	**30**
40	.6777	.9217	1.0850	.7353	20
50	.6799	.9271	1.0786	.7333	10
43° 00′	.6820	.9325	1.0724	.7314	**47° 00′**
10	.6841	.9380	1.0661	.7294	50
20	.6862	.9435	1.0599	.7274	40
30	.6884	.9490	1.0538	.7254	**30**
40	.6905	.9545	1.0477	.7234	20
50	.6926	.9601	1.0416	.7214	10
44° 00′	.6947	.9657	1.0355	.7193	**46° 00′**
10	.6967	.9713	1.0295	.7173	50
20	.6988	.9770	1.0235	.7153	40
30	.7009	.8827	1.0176	.7133	**30**
40	.7030	.9884	1.0117	.7112	20
50	.7050	.9942	1.0058	.7092	10
45° 00′	.7071	1.0000	1.0000	.7071	**45° 00′**
	Cosine	Cotan-gent	Tan-gent	Sine	Angle

Table XI. Logarithms*

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
N	0	1	2	3	4	5	6	7	8	9

* From W. W. Elliott and E. Roy C. Miles. *College Mathematics: A First Course.* New York: Prentice-Hall, Inc., 1940.

Table XI. Logarithms (Cont.)

N	0	1	2	3	4	5	6	7	8	9
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
N	0	1	2	3	4	5	6	7	8	9

Table XI. Logarithms (Cont.)

N	0	1	2	3	4	5	6	7	8	9
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
N	0	1	2	3	4	5	6	7	8	9

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V